FORMALIZED MATHEMATICS
Volume 8, Number 1, 1999
University of Białystok

Homeomorphism between $[:\mathcal{E}^i_{\mathrm{T}}, \mathcal{E}^j_{\mathrm{T}}:]$ and $\mathcal{E}^{i+j}_{\mathrm{T}}$

Artur Korniłowicz[1]
University of Bialystok

Summary. In this paper we introduce the cartesian product of two metric spaces. As the distance between two points in the product we take maximal distance between coordinates of these points. In the main theorem we show the homeomorphism between $[:\mathcal{E}^i_{\mathrm{T}}, \mathcal{E}^j_{\mathrm{T}}:]$ and $\mathcal{E}^{i+j}_{\mathrm{T}}$.

MML Identifier: `TOPREAL7`.

The notation and terminology used in this paper have been introduced in the following articles: [20], [9], [25], [7], [8], [4], [16], [24], [21], [19], [13], [18], [23], [1], [2], [10], [5], [17], [11], [3], [22], [14], [12], [6], [26], and [15].

We use the following convention: i, j, n denote natural numbers, f, g, h, k denote finite sequences of elements of $\mathbb{R}$, and M, N denote non empty metric spaces.

We now state a number of propositions:

(1) For all real numbers a, b such that $\max(a, b) \leqslant a$ holds $\max(a, b) = a$.

(2) For all real numbers a, b, c, d holds $\max(a + c, b + d) \leqslant \max(a, b) + \max(c, d)$.

(3) For all real numbers a, b, c, d, e, f such that $a \leqslant b + c$ and $d \leqslant e + f$ holds $\max(a, d) \leqslant \max(b, e) + \max(c, f)$.

(4) For all finite sequences f, g holds $\mathrm{dom}\, g \subseteq \mathrm{dom}(f \frown g)$.

(5) For all finite sequences f, g such that $\mathrm{len}\, f < i$ and $i \leqslant \mathrm{len}\, f + \mathrm{len}\, g$ holds $i - \mathrm{len}\, f \in \mathrm{dom}\, g$.

(6) For all finite sequences f, g, h, k such that $f \frown g = h \frown k$ and $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$ holds $f = h$ and $g = k$.

(7) If $\mathrm{len}\, f = \mathrm{len}\, g$ or $\mathrm{dom}\, f = \mathrm{dom}\, g$, then $\mathrm{len}(f + g) = \mathrm{len}\, f$ and $\mathrm{dom}(f + g) = \mathrm{dom}\, f$.

[1]This paper was written while the author visited Shinshu University, winter 1999.

ISSN 1426–2630

(8) If $\operatorname{len} f = \operatorname{len} g$ or $\operatorname{dom} f = \operatorname{dom} g$, then $\operatorname{len}(f - g) = \operatorname{len} f$ and $\operatorname{dom}(f - g) = \operatorname{dom} f$.

(9) $\operatorname{len} f = \operatorname{len}^2 f$ and $\operatorname{dom} f = \operatorname{dom}^2 f$.

(10) $\operatorname{len} f = \operatorname{len}|f|$ and $\operatorname{dom} f = \operatorname{dom}|f|$.

(11) ${}^2(f \frown g) = ({}^2 f) \frown ({}^2 g)$.

(12) $|f \frown g| = |f| \frown |g|$.

(13) If $\operatorname{len} f = \operatorname{len} h$ and $\operatorname{len} g = \operatorname{len} k$, then ${}^2(f \frown g + h \frown k) = ({}^2(f + h)) \frown ({}^2(g + k))$.

(14) If $\operatorname{len} f = \operatorname{len} h$ and $\operatorname{len} g = \operatorname{len} k$, then $|f \frown g + h \frown k| = |f + h| \frown |g + k|$.

(15) If $\operatorname{len} f = \operatorname{len} h$ and $\operatorname{len} g = \operatorname{len} k$, then ${}^2(f \frown g - h \frown k) = ({}^2(f - h)) \frown ({}^2(g - k))$.

(16) If $\operatorname{len} f = \operatorname{len} h$ and $\operatorname{len} g = \operatorname{len} k$, then $|f \frown g - h \frown k| = |f - h| \frown |g - k|$.

(17) If $\operatorname{len} f = n$, then $f \in$ the carrier of $\mathcal{E}^n$.

(18) If $\operatorname{len} f = n$, then $f \in$ the carrier of $\mathcal{E}^n_{\mathrm{T}}$.

(19) For every finite sequence f such that $f \in$ the carrier of $\mathcal{E}^n$ holds $\operatorname{len} f = n$.

Let M, N be non empty metric structures. The functor max-Prod2(M, N) yielding a strict metric structure is defined by the conditions (Def. 1).

(Def. 1)(i) The carrier of max-Prod2$(M, N) = [\!:$ the carrier of M, the carrier of $N :\!]$, and

(ii) for all points x, y of max-Prod2(M, N) there exist points x_1, y_1 of M and there exist points x_2, y_2 of N such that $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$ and (the distance of max-Prod2$(M, N))(x, y) = \max(($the distance of $M)(x_1, y_1), ($the distance of $N)(x_2, y_2))$.

Let M, N be non empty metric structures. One can verify that max-Prod2(M, N) is non empty.

Let M, N be non empty metric structures, let x be a point of M, and let y be a point of N. Then $\langle x, y \rangle$ is an element of max-Prod2(M, N).

Let M, N be non empty metric structures and let x be a point of max-Prod2(M, N). Then $x_{\mathbf{1}}$ is an element of M. Then $x_{\mathbf{2}}$ is an element of N.

The following three propositions are true:

(20) Let M, N be non empty metric structures, m_1, m_2 be points of M, and n_1, n_2 be points of N. Then $\rho(\langle m_1, n_1 \rangle, \langle m_2, n_2 \rangle) = \max(\rho(m_1, m_2), \rho(n_1, n_2))$.

(21) For all non empty metric structures M, N and for all points m, n of max-Prod2(M, N) holds $\rho(m, n) = \max(\rho(m_{\mathbf{1}}, n_{\mathbf{1}}), \rho(m_{\mathbf{2}}, n_{\mathbf{2}}))$.

(22) For all Reflexive non empty metric structures M, N holds max-Prod2(M, N) is Reflexive.

Let M, N be Reflexive non empty metric structures. Observe that max-Prod2(M, N) is Reflexive.

Next we state the proposition

(23) For all symmetric non empty metric structures M, N holds max-Prod2(M, N) is symmetric.

Let M, N be symmetric non empty metric structures. Note that max-Prod2(M, N) is symmetric.

Next we state the proposition

(24) For all triangle non empty metric structures M, N holds max-Prod2(M, N) is triangle.

Let M, N be triangle non empty metric structures. One can check that max-Prod2(M, N) is triangle.

Let M, N be non empty metric spaces. Note that max-Prod2(M, N) is discernible.

The following three propositions are true:

(25) $[: M_{\mathrm{top}}, N_{\mathrm{top}} :] = (\text{max-Prod2}(M, N))_{\mathrm{top}}$.

(26) Suppose that

(i) the carrier of $M =$ the carrier of N,

(ii) for every point m of M and for every point n of N and for every real number r such that $r > 0$ and $m = n$ there exists a real number r_1 such that $r_1 > 0$ and $\mathrm{Ball}(n, r_1) \subseteq \mathrm{Ball}(m, r)$, and

(iii) for every point m of M and for every point n of N and for every real number r such that $r > 0$ and $m = n$ there exists a real number r_1 such that $r_1 > 0$ and $\mathrm{Ball}(m, r_1) \subseteq \mathrm{Ball}(n, r)$.

Then $M_{\mathrm{top}} = N_{\mathrm{top}}$.

(27) $[: \mathcal{E}^i_{\mathrm{T}}, \mathcal{E}^j_{\mathrm{T}} :]$ and $\mathcal{E}^{i+j}_{\mathrm{T}}$ are homeomorphic.

Acknowledgments

I would like to thank Professor Yatsuka Nakamura for his help in the preparation of the article.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[3] Leszek Borys. Paracompact and metrizable spaces. *Formalized Mathematics*, 2(**4**):481–485, 1991.

[4] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.

[5] Czesław Byliński. Binary operations applied to finite sequences. *Formalized Mathematics*, 1(**4**):643–649, 1990.

[6] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.

[7] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[8] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[9] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[10] Czesław Byliński. The sum and product of finite sequences of real numbers. *Formalized Mathematics*, 1(**4**):661–668, 1990.
[11] Agata Darmochwał. Families of subsets, subspaces and mappings in topological spaces. *Formalized Mathematics*, 1(**2**):257–261, 1990.
[12] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.
[13] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[14] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.
[15] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.
[16] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.
[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[18] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.
[19] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**):115–122, 1990.
[20] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[21] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[22] Andrzej Trybulec. A Borsuk theorem on homotopy types. *Formalized Mathematics*, 2(**4**):535–545, 1991.
[23] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[24] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[25] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[26] Mariusz Żynel and Adam Guzowski. T_0 topological spaces. *Formalized Mathematics*, 5(**1**):75–77, 1996.

Received February 21, 1999