FORMALIZED MATHEMATICS
Volume 9, Number 1, 2001
University of Białystok

Properties of the External Approximation of Jordan's Curve

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MML Identifier: JORDAN10.

The articles [20], [6], [14], [7], [2], [18], [17], [13], [3], [5], [10], [1], [11], [15], [4], [9], [12], [19], [16], and [8] provide the terminology and notation for this paper.

One can verify that there exists a subset of $\mathcal{E}^2_{\rm T}$ which is connected, compact, non vertical, and non horizontal.

We adopt the following rules: i, j, k, n are natural numbers, P is a subset of $\mathcal{E}^2_{\rm T}$, and C is a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$.

The following propositions are true:

(1) Suppose that

(i) $1 \leqslant k$,

(ii) $k+1 \leqslant \operatorname{len} \operatorname{Cage}(C,n)$,

(iii) $\langle i,\ j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(iv) $\langle i,\ j+1\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(v) $\pi_k \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j}$, and

(vi) $\pi_{k+1} \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j+1}$.

Then $i < \operatorname{len} \operatorname{Gauge}(C,n)$.

(2) Suppose that

(i) $1 \leqslant k$,

(ii) $k+1 \leqslant \operatorname{len} \operatorname{Cage}(C,n)$,

(iii) $\langle i,\ j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(iv) $\langle i,\ j+1\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(v) $\pi_k \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j+1}$, and

(vi) $\pi_{k+1} \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j}$.

Then $i > 1$.

[1]This paper was written while the author visited Shinshu University, winter 1999.

ISSN 1426–2630

(3) Suppose that
(i) $1 \leqslant k$,
(ii) $k+1 \leqslant \operatorname{len} \operatorname{Cage}(C,n)$,
(iii) $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,
(iv) $\langle i+1, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,
(v) $\pi_k \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j}$, and
(vi) $\pi_{k+1} \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i+1,j}$.
Then $j > 1$.

(4) Suppose that
(i) $1 \leqslant k$,
(ii) $k+1 \leqslant \operatorname{len} \operatorname{Cage}(C,n)$,
(iii) $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,
(iv) $\langle i+1, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,
(v) $\pi_k \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i+1,j}$, and
(vi) $\pi_{k+1} \operatorname{Cage}(C,n) = (\operatorname{Gauge}(C,n))_{i,j}$.
Then $j < \operatorname{width} \operatorname{Gauge}(C,n)$.

(5) $C \cap \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)) = \emptyset$.

(6) $\text{N-bound}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)) = \text{N-bound}\, C + \frac{\text{N-bound}\, C - \text{S-bound}\, C}{2^n}$.

(7) If $i < j$, then $\text{N-bound}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,j)) < \text{N-bound}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,i))$.

Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and let n be a natural number. Note that $\overline{\operatorname{RightComp}(\operatorname{Cage}(C,n))}$ is compact.

The following propositions are true:

(8) $\text{N-min}\, C \in \operatorname{RightComp}(\operatorname{Cage}(C,n))$.

(9) $C \cap \operatorname{RightComp}(\operatorname{Cage}(C,n)) \neq \emptyset$.

(10) $C \cap \operatorname{LeftComp}(\operatorname{Cage}(C,n)) = \emptyset$.

(11) $C \subseteq \operatorname{RightComp}(\operatorname{Cage}(C,n))$.

(12) $C \subseteq \operatorname{BDD} \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))$.

(13) $\operatorname{UBD} \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)) \subseteq \operatorname{UBD} C$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. The functor UBD-Family C is defined as follows:

(Def. 1) UBD-Family $C = \{\operatorname{UBD} \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)) : n$ ranges over natural numbers$\}$.

The functor BDD-Family C is defined by:

(Def. 2) BDD-Family $C = \{\operatorname{BDD} \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)) : n$ ranges over natural numbers$\}$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. Then UBD-Family C is a family of subsets of $\mathcal{E}^2_{\mathrm{T}}$. Then BDD-Family C is a family of subsets of $\mathcal{E}^2_{\mathrm{T}}$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$. Note that UBD-Family C is non empty and BDD-Family C is non empty.

One can prove the following propositions:

(14) $\bigcup$UBD-Family C = UBD C.

(15) $C \subseteq \bigcap$BDD-Family C.

(16) BDD $C \cap$ LeftComp(Cage(C, n)) $= \emptyset$.

(17) BDD $C \subseteq$ RightComp(Cage(C, n)).

(18) If P is inside component of C, then $P \cap \widetilde{\mathcal{L}}$(Cage($C, n$)) $= \emptyset$.

(19) BDD $C \cap \widetilde{\mathcal{L}}$(Cage($C, n$)) $= \emptyset$.

(20) $\bigcap$BDD-Family $C = C \cup$ BDD C.

Acknowledgments

I would like to thank Professor Yatsuka Nakamura for his help in the preparation of the article.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[3] Czesław Byliński. Gauges. *Formalized Mathematics*, 8(**1**):25–27, 1999.
[4] Czesław Byliński and Piotr Rudnicki. Bounding boxes for compact sets in $\mathcal{E}^2$. *Formalized Mathematics*, 6(**3**):427–440, 1997.
[5] Czesław Byliński and Mariusz Żynel. Cages - the external approximation of Jordan's curve. *Formalized Mathematics*, 9(**1**):19–24, 2001.
[6] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.
[7] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.
[8] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.
[9] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[10] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.
[11] Rafał Kwiatek. Factorial and Newton coefficients. *Formalized Mathematics*, 1(**5**):887–890, 1990.
[12] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons. Part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.
[13] Yatsuka Nakamura, Andrzej Trybulec, and Czesław Byliński. Bounded domains and unbounded domains. *Formalized Mathematics*, 8(**1**):1–13, 1999.
[14] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.
[15] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[16] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[17] Andrzej Trybulec. Left and right component of the complement of a special closed curve. *Formalized Mathematics*, 5(**4**):465–468, 1996.
[18] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[19] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[20] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

Received June 24, 1999