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On the Isomorphism between Finite Chains

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The notation and terminology used here are introduced in the following papers: [11], [1], [4], [8], [9], [7], [10], [3], [5], [2], and [6].

A relational structure is said to be a chain if:

(Def. 1) It is a connected non empty poset or it is empty.

One can verify that every relational structure which is empty is also reflexive, transitive, and antisymmetric.

One can verify that every chain is reflexive, transitive, and antisymmetric.

Let us note that there exists a chain which is non empty.

One can check that every non empty chain is connected.

Let L be a 1-sorted structure. We say that L is countable if and only if:

(Def. 2) The carrier of L is countable.

Let us observe that there exists a chain which is finite and non empty.

Let us mention that there exists a chain which is countable.

Let A be a connected non empty relational structure. Observe that every non empty relational substructure of A which is full is also connected.

Let A be a finite relational structure. Observe that every relational substructure of A is finite.

We now state the proposition

(1) For all natural numbers n, m such that $n \leqslant m$ holds $\langle n, \subseteq \rangle$ is a full relational substructure of $\langle m, \subseteq \rangle$.

Let L be a relational structure and let A, B be sets. We say that A, B form upper lower partition of L if and only if:

(Def. 3) $A \cup B =$ the carrier of L and for all elements a, b of L such that $a \in A$ and $b \in B$ holds $a < b$.

Next we state four propositions:

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(2) Let L be a relational structure and A, B be sets. If A, B form upper lower partition of L, then $A \cap B = \emptyset$.

(3) Let L be an upper-bounded antisymmetric non empty relational structure. Then (the carrier of L) $\setminus \{\top_L\}$, $\{\top_L\}$ form upper lower partition of L.

(4) Let L_1, L_2 be relational structures and f be a map from L_1 into L_2. Suppose f is isomorphic. Then

(i) the carrier of $L_1 \neq \emptyset$ iff the carrier of $L_2 \neq \emptyset$,

(ii) the carrier of $L_2 \neq \emptyset$ or the carrier of $L_1 = \emptyset$, and

(iii) the carrier of $L_1 = \emptyset$ iff the carrier of $L_2 = \emptyset$.

(5) Let L_1, L_2 be antisymmetric relational structures and A_1, B_1 be subsets of L_1. Suppose A_1, B_1 form upper lower partition of L_1. Let A_2, B_2 be subsets of L_2. Suppose A_2, B_2 form upper lower partition of L_2. Let f be a map from $\mathrm{sub}(A_1)$ into $\mathrm{sub}(A_2)$. Suppose f is isomorphic. Let g be a map from $\mathrm{sub}(B_1)$ into $\mathrm{sub}(B_2)$. Suppose g is isomorphic. Then there exists a map h from L_1 into L_2 such that $h = f {+\cdot} g$ and h is isomorphic.

Let n be a natural number. Observe that $n + 1$ is non empty.

The following proposition is true

(6) Let A be a finite chain and n be a natural number. If $\overline{\overline{\text{the carrier of } A}} = n$, then A and $\langle n, \subseteq \rangle$ are isomorphic.

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