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Some Properties of Cells and Arcs[1]

Robert Milewski
University of Białystok

Andrzej Trybulec
University of Białystok

Artur Korniłowicz
University of Białystok

Adam Naumowicz
University of Białystok

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The notation and terminology used in this paper are introduced in the following papers: [25], [2], [11], [26], [21], [12], [3], [5], [30], [7], [28], [6], [18], [22], [17], [24], [20], [23], [8], [10], [16], [1], [27], [9], [4], [15], [32], [19], [29], [31], [13], and [14].

For simplicity, we adopt the following convention: E denotes a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$, C denotes a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$, G denotes a Go-board, i, j, m, n denote natural numbers, and p denotes a point of $\mathcal{E}^2_{\mathrm{T}}$.

Let us observe that every simple closed curve is non vertical and non horizontal.

Let T be a non empty topological space. Note that there exists a union of components of T which is non empty.

The following propositions are true:

(1) Let T be a non empty topological space and A be a non empty union of components of T. If A is connected, then A is a component of T.

(2) For every finite sequence f holds f is empty iff $\mathrm{Rev}(f)$ is empty.

(3) Let D be a non empty set, f be a finite sequence of elements of D, and given i, j. If $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$, then $\mathrm{mid}(f, i, j)$ is non empty.

(4) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If $1 \leqslant \mathrm{len}\, f$ and $p \in \widetilde{\mathcal{L}}(f)$, then $(\downharpoonleft f, p)(1) = f(1)$.

(5) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If f is a special sequence and $p \in \widetilde{\mathcal{L}}(f)$, then $(\downharpoonleft p, f)(\mathrm{len} \downharpoonleft p, f) = f(\mathrm{len}\, f)$.

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(6) For every simple closed curve P holds W-max $P \neq$ E-max P.

(7) Let D be a non empty set and f be a finite sequence of elements of D. If $1 \leqslant i$ and $i < \operatorname{len} f$, then $(\operatorname{mid}(f, i, \operatorname{len} f -' 1)) \frown \langle f_{\operatorname{len} f} \rangle = \operatorname{mid}(f, i, \operatorname{len} f)$.

(8) For all points p, q of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \neq q$ and $\mathcal{L}(p, q)$ is vertical holds $\langle p, q \rangle$ is a special sequence.

(9) For all points p, q of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \neq q$ and $\mathcal{L}(p, q)$ is horizontal holds $\langle p, q \rangle$ is a special sequence.

(10) Let p, q be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$ and v be a point of $\mathcal{E}^2_{\mathrm{T}}$. If p is in the area of q, then $p^{v}_{\circlearrowleft}$ is in the area of q.

(11) For every non trivial finite sequence p of elements of $\mathcal{E}^2_{\mathrm{T}}$ and for every point v of $\mathcal{E}^2_{\mathrm{T}}$ holds $p^{v}_{\circlearrowleft}$ is in the area of p.

(12) For every finite sequence f holds Center $f \geqslant 1$.

(13) For every finite sequence f such that $\operatorname{len} f \geqslant 1$ holds Center $f \leqslant \operatorname{len} f$.

(14) Center $G \leqslant \operatorname{len} G$.

(15) For every finite sequence f such that $\operatorname{len} f \geqslant 2$ holds Center $f > 1$.

(16) For every finite sequence f such that $\operatorname{len} f \geqslant 3$ holds Center $f < \operatorname{len} f$.

(17) Center Gauge$(E, n) = 2^{n -' 1} + 2$.

(18) $E \subseteq \operatorname{cell}(\operatorname{Gauge}(E, 0), 2, 2)$.

(19) $\operatorname{cell}(\operatorname{Gauge}(E, 0), 2, 2) \not\subseteq \operatorname{BDD} E$.

(20) $(\operatorname{Gauge}(C, 1))_{\text{Center Gauge}(C,1),1} = [\frac{\text{W-bound}\, C + \text{E-bound}\, C}{2}, \text{S-bound}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, 1))]$.

(21) $(\operatorname{Gauge}(C, 1))_{\text{Center Gauge}(C,1),\operatorname{len} \operatorname{Gauge}(C,1)} = [\frac{\text{W-bound}\, C + \text{E-bound}\, C}{2}, \text{N-bound}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, 1))]$.

(22) If $1 \leqslant j$ and $j < \operatorname{width} G$ and $1 \leqslant m$ and $m \leqslant \operatorname{len} G$ and $1 \leqslant n$ and $n \leqslant \operatorname{width} G$ and $p \in \operatorname{cell}(G, \operatorname{len} G, j)$ and $p_{\mathbf{1}} = (G_{m,n})_{\mathbf{1}}$, then $\operatorname{len} G = m$.

(23) Suppose $1 \leqslant i$ and $i \leqslant \operatorname{len} G$ and $1 \leqslant j$ and $j < \operatorname{width} G$ and $1 \leqslant m$ and $m \leqslant \operatorname{len} G$ and $1 \leqslant n$ and $n \leqslant \operatorname{width} G$ and $p \in \operatorname{cell}(G, i, j)$ and $p_{\mathbf{1}} = (G_{m,n})_{\mathbf{1}}$. Then $i = m$ or $i = m -' 1$.

(24) If $1 \leqslant i$ and $i < \operatorname{len} G$ and $1 \leqslant m$ and $m \leqslant \operatorname{len} G$ and $1 \leqslant n$ and $n \leqslant \operatorname{width} G$ and $p \in \operatorname{cell}(G, i, \operatorname{width} G)$ and $p_{\mathbf{2}} = (G_{m,n})_{\mathbf{2}}$, then $\operatorname{width} G = n$.

(25) Suppose $1 \leqslant i$ and $i < \operatorname{len} G$ and $1 \leqslant j$ and $j \leqslant \operatorname{width} G$ and $1 \leqslant m$ and $m \leqslant \operatorname{len} G$ and $1 \leqslant n$ and $n \leqslant \operatorname{width} G$ and $p \in \operatorname{cell}(G, i, j)$ and $p_{\mathbf{2}} = (G_{m,n})_{\mathbf{2}}$. Then $j = n$ or $j = n -' 1$.

(26) For every simple closed curve C and for every real number r such that W-bound $C \leqslant r$ and $r \leqslant$ E-bound C holds $\mathcal{L}([r, \text{S-bound}\, C], [r, \text{N-bound}\, C])$ meets UpperArc C.

(27) For every simple closed curve C and for every real number r such that W-bound $C \leqslant r$ and $r \leqslant$ E-bound C holds $\mathcal{L}([r, \text{S-bound}\, C], [r,

N-bound C]) meets LowerArc C.

(28) Let C be a simple closed curve and i be a natural number. If $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C, n)$, then $\mathcal{L}((\operatorname{Gauge}(C, n))_{i,1}, (\operatorname{Gauge}(C, n))_{i, \operatorname{len} \operatorname{Gauge}(C,n)})$ meets UpperArc C.

(29) Let C be a simple closed curve and i be a natural number. If $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C, n)$, then $\mathcal{L}((\operatorname{Gauge}(C, n))_{i,1}, (\operatorname{Gauge}(C, n))_{i, \operatorname{len} \operatorname{Gauge}(C,n)})$ meets LowerArc C.

(30) For every simple closed curve C holds $\mathcal{L}((\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n),1}, (\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)})$ meets UpperArc C.

(31) For every simple closed curve C holds $\mathcal{L}((\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n),1}, (\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)})$ meets LowerArc C.

(32) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. If $1 \leqslant i$ and $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\mathcal{L}((\operatorname{Gauge}(C, n))_{i,1}, (\operatorname{Gauge}(C, n))_{i, \operatorname{len} \operatorname{Gauge}(C,n)})$ meets UpperArc $\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n))$.

(33) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. If $1 \leqslant i$ and $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\mathcal{L}((\operatorname{Gauge}(C, n))_{i,1}, (\operatorname{Gauge}(C, n))_{i, \operatorname{len} \operatorname{Gauge}(C,n)})$ meets LowerArc $\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n))$.

(34) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathcal{L}((\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n),1}, (\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)})$ meets UpperArc $\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n))$.

(35) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathcal{L}((\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n),1}, (\operatorname{Gauge}(C, n))_{\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)})$ meets LowerArc $\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n))$.

(36) If $j \leqslant \operatorname{width} G$, then $\operatorname{cell}(G, 0, j)$ is not Bounded.

(37) If $i \leqslant \operatorname{width} G$, then $\operatorname{cell}(G, \operatorname{len} G, i)$ is not Bounded.

(38) If $j \leqslant \operatorname{width} \operatorname{Gauge}(C, n)$, then $\operatorname{cell}(\operatorname{Gauge}(C, n), 0, j) \subseteq \operatorname{UBD} C$.

(39) If $j \leqslant \operatorname{len} \operatorname{Gauge}(E, n)$, then $\operatorname{cell}(\operatorname{Gauge}(E, n), \operatorname{len} \operatorname{Gauge}(E, n), j) \subseteq \operatorname{UBD} E$.

(40) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$ and $j \leqslant \operatorname{width} \operatorname{Gauge}(C, n)$ and $\operatorname{cell}(\operatorname{Gauge}(C, n), i, j) \subseteq \operatorname{BDD} C$, then $j > 1$.

(41) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$ and $j \leqslant \operatorname{width} \operatorname{Gauge}(C, n)$ and $\operatorname{cell}(\operatorname{Gauge}(C, n), i, j) \subseteq \operatorname{BDD} C$, then $i > 1$.

(42) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$ and $j \leqslant \operatorname{width} \operatorname{Gauge}(C, n)$ and $\operatorname{cell}(\operatorname{Gauge}(C, n), i, j) \subseteq \operatorname{BDD} C$, then $j + 1 < \operatorname{width} \operatorname{Gauge}(C, n)$.

(43) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$ and $j \leqslant \operatorname{width} \operatorname{Gauge}(C, n)$ and $\operatorname{cell}(\operatorname{Gauge}(C, n), i, j) \subseteq \operatorname{BDD} C$, then $i + 1 < \operatorname{len} \operatorname{Gauge}(C, n)$.

(44) If there exist i, j such that $i \leqslant \operatorname{len}\operatorname{Gauge}(C,n)$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C,n)$ and $\operatorname{cell}(\operatorname{Gauge}(C,n),i,j) \subseteq \operatorname{BDD} C$, then $n \geqslant 1$.

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