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Some Remarks on Finite Sequences on Go-boards[1]

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Summary. This paper shows some properties of finite sequences on Go-boards. It also provides the partial correspondence between two ways of decomposition of curves induced by cages.

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The articles [20], [24], [8], [19], [9], [2], [3], [22], [4], [15], [14], [16], [18], [5], [7], [13], [1], [6], [12], [17], [23], [21], [10], and [11] provide the terminology and notation for this paper.

We follow the rules: i, j, k, n denote natural numbers, f denotes a finite sequence of elements of the carrier of $\mathcal{E}^2_{\rm T}$, and G denotes a Go-board.

We now state several propositions:

(1) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (i,j), G \circ (i,k))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle i,\, j\rangle \in$ the indices of G,

(iv) $\langle i,\, k\rangle \in$ the indices of G, and

(v) $j \leqslant k$.

Then there exists n such that $j \leqslant n$ and $n \leqslant k$ and $(G \circ (i,n))_{\mathbf{2}} = \inf(\mathrm{proj2}^\circ(\mathcal{L}(G \circ (i,j), G \circ (i,k)) \cap \widetilde{\mathcal{L}}(f)))$.

(2) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (i,j), G \circ (i,k))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle i,\, j\rangle \in$ the indices of G,

(iv) $\langle i,\, k\rangle \in$ the indices of G, and

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(v) $j \leqslant k$.

Then there exists n such that $j \leqslant n$ and $n \leqslant k$ and $(G \circ (i, n))_{\mathbf{2}} = \sup(\mathrm{proj2}^{\circ}(\mathcal{L}(G \circ (i, j), G \circ (i, k)) \cap \widetilde{\mathcal{L}}(f)))$.

(3) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (j, i), G \circ (k, i))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle j, i \rangle \in$ the indices of G,

(iv) $\langle k, i \rangle \in$ the indices of G, and

(v) $j \leqslant k$.

Then there exists n such that $j \leqslant n$ and $n \leqslant k$ and $(G \circ (n, i))_{\mathbf{1}} = \inf(\mathrm{proj1}^{\circ}(\mathcal{L}(G \circ (j, i), G \circ (k, i)) \cap \widetilde{\mathcal{L}}(f)))$.

(4) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (j, i), G \circ (k, i))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle j, i \rangle \in$ the indices of G,

(iv) $\langle k, i \rangle \in$ the indices of G, and

(v) $j \leqslant k$.

Then there exists n such that $j \leqslant n$ and $n \leqslant k$ and $(G \circ (n, i))_{\mathbf{1}} = \sup(\mathrm{proj1}^{\circ}(\mathcal{L}(G \circ (j, i), G \circ (k, i)) \cap \widetilde{\mathcal{L}}(f)))$.

(5) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{UpperSeq}(C, n))_1 = \text{W-min}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$.

(6) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{LowerSeq}(C, n))_1 = \text{E-max}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$.

(7) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{UpperSeq}(C, n))_{\mathrm{len}\,\mathrm{UpperSeq}(C,n)} = \text{E-max}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$.

(8) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{LowerSeq}(C, n))_{\mathrm{len}\,\mathrm{LowerSeq}(C,n)} = \text{W-min}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$.

(9) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C, n)) = \mathrm{UpperArc}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$ and $\widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C, n)) = \mathrm{LowerArc}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$ or $\widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C, n)) = \mathrm{LowerArc}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$ and $\widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C, n)) = \mathrm{UpperArc}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C, n))$.

We adopt the following convention: C is a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ satisfying conditions of simple closed curve, p is a point of $\mathcal{E}^2_{\mathrm{T}}$, and i_1, j_1, i_2, j_2 are natural numbers.

Next we state four propositions:

(10) Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\mathrm{UpperSeq}(C, n)$ is a sequence which elements belong to $\mathrm{Gauge}(C, n)$.

(11) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) f is a sequence which elements belong to G,

(ii) there exist i, j such that $\langle i,\, j\rangle \in$ the indices of G and $p = G \circ (i,j)$, and

(iii) for all i_1, j_1, i_2, j_2 such that $\langle i_1,\, j_1\rangle \in$ the indices of G and $\langle i_2,\, j_2\rangle \in$ the indices of G and $p = G\circ(i_1, j_1)$ and $f_1 = G\circ(i_2, j_2)$ holds $|i_2-i_1|+|j_2-j_1| = 1$.

Then $\langle p\rangle \frown f$ is a sequence which elements belong to G.

(12) Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$ and n be a natural number. Then LowerSeq(C, n) is a sequence which elements belong to Gauge(C, n).

(13) Suppose $p_{\mathbf{1}} = \frac{\text{W-bound}\, C + \text{E-bound}\, C}{2}$ and $p_{\mathbf{2}} = \inf(\text{proj}2^{\circ}(\mathcal{L}(\text{Gauge}(C,1) \circ (\text{Center Gauge}(C,1),1), \text{Gauge}(C,1) \circ (\text{Center Gauge}(C,1), \text{width Gauge}(C,1))) \cap \text{UpperArc}\, \widetilde{\mathcal{L}}(\text{Cage}(C, i+1))))$. Then there exists j such that $1 \leqslant j$ and $j \leqslant \text{width Gauge}(C, i+1)$ and $p = \text{Gauge}(C, i+1) \circ (\text{Center Gauge}(C, i+1), j)$.

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