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Combining of Multi Cell Circuits

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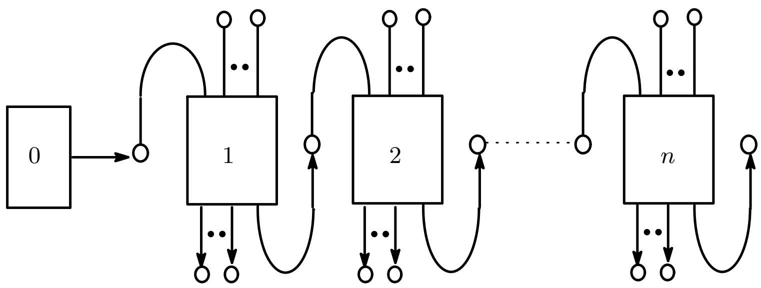
Summary. In this article we continue the investigations from [11] and [3] of verification of a circuit design. We concentrate on the combination of multi cell circuits from given cells (circuit modules). Namely, we formalize a design of the form



and prove its stability. The formalization proposed consists in a series of schemes which allow to define multi cells circuits and prove their properties. Our goal is to achive mathematical formalization which will allow to verify designs of real circuits.

MML Identifier: `CIRCCMB2`.

The articles [18], [2], [11], [12], [13], [3], [4], [9], [5], [6], [7], [10], [14], [16], [1], [8], [19], [20], [17], and [15] provide the terminology and notation for this paper.

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1. One Gate Circuits

Let n be a natural number, let f be a function from $Boolean^n$ into $Boolean$, and let p be a finite sequence with length n. One can verify that $\mathrm{1GateCircuit}(p, f)$ is Boolean.

The following four propositions are true:

(1) Let X be a finite non empty set, n be a natural number, p be a finite sequence with length n, f be a function from X^n into X, o be an operation symbol of $\mathrm{1GateCircStr}(p, f)$, and s be a state of $\mathrm{1GateCircuit}(p, f)$. Then o depends-on-in $s = s \cdot p$.

(2) Let X be a finite non empty set, n be a natural number, p be a finite sequence with length n, f be a function from X^n into X, and s be a state of $\mathrm{1GateCircuit}(p, f)$. Then $\mathrm{Following}(s)$ is stable.

(3) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stable, then for every natural number n holds $\mathrm{Following}(s, n) = s$.

(4) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n_1, n_2 be natural numbers. If $\mathrm{Following}(s, n_1)$ is stable and $n_1 \leqslant n_2$, then $\mathrm{Following}(s, n_2) = \mathrm{Following}(s, n_1)$.

2. Defining Multi Cell Circuit Structures

In this article we present several logical schemes. The scheme *CIRCCMB2'sch 1* deals with a non empty many sorted signature $\mathcal{A}$, a set $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, and a binary functor $\mathcal{G}$ yielding a set, and states that:

There exist many sorted sets f, h indexed by $\mathbb{N}$ such that

(i) $f(0) = \mathcal{A}$,

(ii) $h(0) = \mathcal{B}$, and

(iii) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$

for all values of the parameters.

The scheme *CIRCCMB2'sch 2* deals with a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, many sorted sets $\mathcal{A}$, $\mathcal{B}$ indexed by $\mathbb{N}$, and a ternary predicate $\mathcal{P}$, and states that:

For every natural number n there exists a non empty many sorted signature S such that $S = \mathcal{A}(n)$ and $\mathcal{P}[S, \mathcal{B}(n), n]$

provided the parameters meet the following requirements:

- There exists a non empty many sorted signature S and there exists a set x such that $S = \mathcal{A}(0)$ and $x = \mathcal{B}(0)$ and $\mathcal{P}[S, x, 0]$,
- Let n be a natural number, S be a non empty many sorted signature, and x be a set. If $S = \mathcal{A}(n)$ and $x = \mathcal{B}(n)$, then $\mathcal{A}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{B}(n+1) = \mathcal{G}(x, n)$, and
- Let n be a natural number, S be a non empty many sorted signature, and x be a set. If $S = \mathcal{A}(n)$ and $x = \mathcal{B}(n)$ and $\mathcal{P}[S, x, n]$, then $\mathcal{P}[\mathcal{F}(S, x, n), \mathcal{G}(x, n), n+1]$.

The scheme *CIRCCMB2'sch 3* deals with a non empty many sorted signature $\mathcal{A}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, and many sorted sets $\mathcal{B}$, $\mathcal{C}$ indexed by $\mathbb{N}$, and states that:

For every natural number n and for every set x such that $x = \mathcal{C}(n)$ holds $\mathcal{C}(n+1) = \mathcal{G}(x, n)$

provided the following requirements are met:

- $\mathcal{B}(0) = \mathcal{A}$, and
- Let n be a natural number, S be a non empty many sorted signature, and x be a set. If $S = \mathcal{B}(n)$ and $x = \mathcal{C}(n)$, then $\mathcal{B}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{C}(n+1) = \mathcal{G}(x, n)$.

The scheme *CIRCCMB2'sch 4* deals with a non empty many sorted signature $\mathcal{A}$, a set $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

There exists a non empty many sorted signature S and there exist many sorted sets f, h indexed by $\mathbb{N}$ such that

(i) $S = f(\mathcal{C})$,

(ii) $f(0) = \mathcal{A}$,

(iii) $h(0) = \mathcal{B}$, and

(iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$

for all values of the parameters.

The scheme *CIRCCMB2'sch 5* deals with a non empty many sorted signature $\mathcal{A}$, a set $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

Let S_1, S_2 be non empty many sorted signatures. Suppose that

(i) there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_1 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$, and

(ii) there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_2 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$.

Then $S_1 = S_2$

for all values of the parameters.

The scheme *CIRCCMB2'sch 6* deals with a non empty many sorted signature $\mathcal{A}$, a set $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

(i) There exists a non empty many sorted signature S and there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$, and

(ii) for all non empty many sorted signatures S_1, S_2 such that there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_1 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$ and there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_2 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$ holds $S_1 = S_2$

for all values of the parameters.

The scheme *CIRCCMB2'sch 7* deals with a non empty many sorted signature $\mathcal{A}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a set $\mathcal{B}$, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

There exists an unsplit non void non empty non empty strict many sorted signature S with arity held in gates and Boolean denotation held in gates and there exist many sorted sets f, h indexed by $\mathbb{N}$ such that

(i) $S = f(\mathcal{C})$,

(ii) $f(0) = \mathcal{A}$,

(iii) $h(0) = \mathcal{B}$, and

(iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$

provided the parameters meet the following requirements:

- $\mathcal{A}$ is unsplit, non void, non empty, and strict and has arity held in gates and Boolean denotation held in gates, and

- Let S be an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates, x be a set, and n be a natural number. Then $\mathcal{F}(S, x, n)$ is unsplit, non void, non empty, and strict and has arity held in gates and Boolean denotation held in gates.

The scheme *CIRCCMB2'sch 8* deals with a non empty many sorted signature $\mathcal{A}$, a binary functor $\mathcal{F}$ yielding an unsplit non void non empty many sorted signature with arity held in gates and Boolean denotation held in gates, a set $\mathcal{B}$, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

There exists an unsplit non void non empty non empty strict many sorted signature S with arity held in gates and Boolean denotation held in gates and there exist many sorted sets f, h indexed by $\mathbb{N}$ such that

(i) $S = f(\mathcal{C})$,

(ii) $f(0) = \mathcal{A}$,

(iii) $h(0) = \mathcal{B}$, and

(iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = S +\cdot \mathcal{F}(x, n)$ and $h(n+1) = \mathcal{G}(x, n)$

provided the parameters meet the following requirement:

- $\mathcal{A}$ is unsplit, non void, non empty, and strict and has arity held in gates and Boolean denotation held in gates.

The scheme *CIRCCMB2'sch 9* deals with a non empty many sorted signature $\mathcal{A}$, a set $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a binary functor $\mathcal{G}$ yielding a set, and a natural number $\mathcal{C}$, and states that:

Let S_1, S_2 be unsplit non void non empty strict non empty many sorted signatures with arity held in gates and Boolean denotation held in gates. Suppose that

(i) there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_1 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$, and

(ii) there exist many sorted sets f, h indexed by $\mathbb{N}$ such that $S_2 = f(\mathcal{C})$ and $f(0) = \mathcal{A}$ and $h(0) = \mathcal{B}$ and for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{G}(x, n)$.

Then $S_1 = S_2$

for all values of the parameters.

3. Input of Multi Cell Circuit

We now state several propositions:

(5) For all functions f, g such that $f \approx g$ holds $\mathrm{rng}(f{+}{\cdot}g) = \mathrm{rng}\, f \cup \mathrm{rng}\, g$.

(6) For all non empty many sorted signatures S_1, S_2 such that $S_1 \approx S_2$ holds $\mathrm{InputVertices}(S_1{+}{\cdot}S_2) = (\mathrm{InputVertices}(S_1) \setminus \mathrm{InnerVertices}(S_2)) \cup (\mathrm{InputVertices}(S_2) \setminus \mathrm{InnerVertices}(S_1))$.

(7) For every set X with no pairs and for every binary relation Y holds $X \setminus Y = X$.

(8) For every binary relation X and for all sets Y, Z such that $Z \subseteq Y$ and $Y \setminus Z$ has no pairs holds $X \setminus Y = X \setminus Z$.

(9) For all sets X, Z and for every binary relation Y such that $Z \subseteq Y$ and $X \setminus Z$ has no pairs holds $X \setminus Y = X \setminus Z$.

Now we present two schemes. The scheme *CIRCCMB2'sch 10* deals with an unsplit non void non empty many sorted signature $\mathcal{A}$ with arity held in gates and Boolean denotation held in gates, a unary functor $\mathcal{F}$ yielding a set, a many sorted set $\mathcal{B}$ indexed by $\mathbb{N}$, a binary functor $\mathcal{G}$ yielding an unsplit non void non empty many sorted signature with arity held in gates and Boolean denotation held in gates, and a binary functor $\mathcal{H}$ yielding a set, and states that:

> Let n be a natural number. Then there exist unsplit non void non empty many sorted signatures S_1, S_2 with arity held in gates and Boolean denotation held in gates such that $S_1 = \mathcal{F}(n)$ and $S_2 = \mathcal{F}(n+1)$ and $\mathrm{InputVertices}(S_2) = \mathrm{InputVertices}(S_1) \cup (\mathrm{InputVertices}(\mathcal{G}(\mathcal{B}(n), n)) \setminus \{\mathcal{B}(n)\})$ and $\mathrm{InnerVertices}(S_1)$ is a binary relation and $\mathrm{InputVertices}(S_1)$ has no pairs

provided the following requirements are met:

- $\mathrm{InnerVertices}(\mathcal{A})$ is a binary relation,
- $\mathrm{InputVertices}(\mathcal{A})$ has no pairs,
- $\mathcal{F}(0) = \mathcal{A}$ and $\mathcal{B}(0) \in \mathrm{InnerVertices}(\mathcal{A})$,
- For every natural number n and for every set x holds $\mathrm{InnerVertices}(\mathcal{G}(x, n))$ is a binary relation,
- For every natural number n and for every set x such that $x = \mathcal{B}(n)$ holds $\mathrm{InputVertices}(\mathcal{G}(x, n)) \setminus \{x\}$ has no pairs, and
- Let n be a natural number, S be a non empty many sorted signature, and x be a set. Suppose $S = \mathcal{F}(n)$ and $x = \mathcal{B}(n)$. Then $\mathcal{F}(n+1) = S{+}{\cdot}\mathcal{G}(x, n)$ and $\mathcal{B}(n+1) = \mathcal{H}(x, n)$ and $x \in \mathrm{InputVertices}(\mathcal{G}(x, n))$ and $\mathcal{H}(x, n) \in \mathrm{InnerVertices}(\mathcal{G}(x, n))$.

The scheme *CIRCCMB2'sch 11* deals with a unary functor $\mathcal{F}$ yielding an unsplit non void non empty many sorted signature with arity held in gates and Boolean denotation held in gates, a many sorted set $\mathcal{A}$ indexed by $\mathbb{N}$, a binary functor $\mathcal{G}$ yielding an unsplit non void non empty many sorted signature with arity held in gates and Boolean denotation held in gates, and a binary functor $\mathcal{H}$ yielding a set, and states that:

For every natural number n holds $\mathrm{InputVertices}(\mathcal{F}(n+1)) = \mathrm{InputVertices}(\mathcal{F}(n)) \cup (\mathrm{InputVertices}(\mathcal{G}(\mathcal{A}(n), n)) \setminus \{\mathcal{A}(n)\})$ and $\mathrm{InnerVertices}(\mathcal{F}(n))$ is a binary relation and $\mathrm{InputVertices}(\mathcal{F}(n))$ has no pairs

provided the parameters meet the following requirements:

- $\mathrm{InnerVertices}(\mathcal{F}(0))$ is a binary relation,
- $\mathrm{InputVertices}(\mathcal{F}(0))$ has no pairs,
- $\mathcal{A}(0) \in \mathrm{InnerVertices}(\mathcal{F}(0))$,
- For every natural number n and for every set x holds $\mathrm{InnerVertices}(\mathcal{G}(x, n))$ is a binary relation,
- For every natural number n and for every set x such that $x = \mathcal{A}(n)$ holds $\mathrm{InputVertices}(\mathcal{G}(x, n)) \setminus \{x\}$ has no pairs, and
- Let n be a natural number, S be a non empty many sorted signature, and x be a set. Suppose $S = \mathcal{F}(n)$ and $x = \mathcal{A}(n)$. Then $\mathcal{F}(n+1) = S +\cdot \mathcal{G}(x, n)$ and $\mathcal{A}(n+1) = \mathcal{H}(x, n)$ and $x \in \mathrm{InputVertices}(\mathcal{G}(x, n))$ and $\mathcal{H}(x, n) \in \mathrm{InnerVertices}(\mathcal{G}(x, n))$.

4. Defining Multi Cell Circuits

Now we present several schemes. The scheme *CIRCCMB2'sch 12* deals with a non empty many sorted signature $\mathcal{A}$, a non-empty algebra $\mathcal{B}$ over $\mathcal{A}$, a set $\mathcal{C}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, and a binary functor $\mathcal{H}$ yielding a set, and states that:

There exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that

(i) $f(0) = \mathcal{A}$,

(ii) $g(0) = \mathcal{B}$,

(iii) $h(0) = \mathcal{C}$, and

(iv) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$

for all values of the parameters.

The scheme *CIRCCMB2'sch 13* deals with a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, many sorted sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ indexed by $\mathbb{N}$, and a 4-ary predicate $\mathcal{P}$, and states that:

Let n be a natural number. Then there exists a non empty many sorted signature S and there exists a non-empty algebra A over S such that $S = \mathcal{A}(n)$ and $A = \mathcal{B}(n)$ and $\mathcal{P}[S, A, \mathcal{C}(n), n]$

provided the following conditions are satisfied:

- There exists a non empty many sorted signature S and there exists a non-empty algebra A over S and there exists a set x such that $S = \mathcal{A}(0)$ and $A = \mathcal{B}(0)$ and $x = \mathcal{C}(0)$ and $\mathcal{P}[S, A, x, 0]$,
- Let n be a natural number, S be a non empty many sorted signature, A be a non-empty algebra over S, and x be a set. Suppose $S = \mathcal{A}(n)$ and $A = \mathcal{B}(n)$ and $x = \mathcal{C}(n)$. Then $\mathcal{A}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{B}(n+1) = \mathcal{G}(S, A, x, n)$ and $\mathcal{C}(n+1) = \mathcal{H}(x, n)$,
- Let n be a natural number, S be a non empty many sorted signature, A be a non-empty algebra over S, and x be a set. If $S = \mathcal{A}(n)$ and $A = \mathcal{B}(n)$ and $x = \mathcal{C}(n)$ and $\mathcal{P}[S, A, x, n]$, then $\mathcal{P}[\mathcal{F}(S, x, n), \mathcal{G}(S, A, x, n), \mathcal{H}(x, n), n+1]$, and
- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 14* deals with a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and many sorted sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$ indexed by $\mathbb{N}$, and states that:

$\mathcal{A} = \mathcal{B}$ and $\mathcal{C} = \mathcal{D}$ and $\mathcal{E} = \mathcal{F}$

provided the parameters meet the following conditions:

- There exists a non empty many sorted signature S and there exists a non-empty algebra A over S such that $S = \mathcal{A}(0)$ and $A = \mathcal{C}(0)$,
- $\mathcal{A}(0) = \mathcal{B}(0)$ and $\mathcal{C}(0) = \mathcal{D}(0)$ and $\mathcal{E}(0) = \mathcal{F}(0)$,
- Let n be a natural number, S be a non empty many sorted signature, A be a non-empty algebra over S, and x be a set. Suppose $S = \mathcal{A}(n)$ and $A = \mathcal{C}(n)$ and $x = \mathcal{E}(n)$. Then $\mathcal{A}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{C}(n+1) = \mathcal{G}(S, A, x, n)$ and $\mathcal{E}(n+1) = \mathcal{H}(x, n)$,
- Let n be a natural number, S be a non empty many sorted signature, A be a non-empty algebra over S, and x be a set. Suppose $S = \mathcal{B}(n)$ and $A = \mathcal{D}(n)$ and $x = \mathcal{F}(n)$. Then $\mathcal{B}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{D}(n+1) = \mathcal{G}(S, A, x, n)$ and $\mathcal{F}(n+1) = \mathcal{H}(x, n)$, and
- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 15* deals with a non empty many sorted signature $\mathcal{A}$, a non-empty algebra $\mathcal{B}$ over $\mathcal{A}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and many sorted sets $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ indexed by $\mathbb{N}$, and states that:

Let n be a natural number, S be a non empty many sorted signature, and x be a set. If $S = \mathcal{C}(n)$ and $x = \mathcal{E}(n)$, then

$\mathcal{C}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{E}(n+1) = \mathcal{H}(x, n)$

provided the parameters meet the following conditions:

- $\mathcal{C}(0) = \mathcal{A}$ and $\mathcal{D}(0) = \mathcal{B}$,
- Let n be a natural number, S be a non empty many sorted signature, A be a non-empty algebra over S, and x be a set. Suppose $S = \mathcal{C}(n)$ and $A = \mathcal{D}(n)$ and $x = \mathcal{E}(n)$. Then $\mathcal{C}(n+1) = \mathcal{F}(S, x, n)$ and $\mathcal{D}(n+1) = \mathcal{G}(S, A, x, n)$ and $\mathcal{E}(n+1) = \mathcal{H}(x, n)$, and
- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 16* deals with a non empty many sorted signature $\mathcal{A}$, a non-empty algebra $\mathcal{B}$ over $\mathcal{A}$, a set $\mathcal{C}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{D}$, and states that:

There exists a non empty many sorted signature S and there exists a non-empty algebra A over S and there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that

(i) $S = f(\mathcal{D})$,
(ii) $A = g(\mathcal{D})$,
(iii) $f(0) = \mathcal{A}$,
(iv) $g(0) = \mathcal{B}$,
(v) $h(0) = \mathcal{C}$, and
(vi) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$

provided the following condition is satisfied:

- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 17* deals with non empty many sorted signatures $\mathcal{A}$, $\mathcal{B}$, a non-empty algebra $\mathcal{C}$ over $\mathcal{A}$, a set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{E}$, and states that:

There exists a non-empty algebra A over $\mathcal{B}$ and there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that

(i) $\mathcal{B} = f(\mathcal{E})$,
(ii) $A = g(\mathcal{E})$,
(iii) $f(0) = \mathcal{A}$,
(iv) $g(0) = \mathcal{C}$,
(v) $h(0) = \mathcal{D}$, and

(vi) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$

provided the parameters meet the following requirements:

- There exist many sorted sets f, h indexed by $\mathbb{N}$ such that
  (i) $\mathcal{B} = f(\mathcal{E})$,
  (ii) $f(0) = \mathcal{A}$,
  (iii) $h(0) = \mathcal{D}$, and
  (iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$,
  and
- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 18* deals with non empty many sorted signatures $\mathcal{A}$, $\mathcal{B}$, a non-empty algebra $\mathcal{C}$ over $\mathcal{A}$, a set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{E}$, and states that:

Let A_1, A_2 be non-empty algebras over $\mathcal{B}$. Suppose that
(i) there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that $\mathcal{B} = f(\mathcal{E})$ and $A_1 = g(\mathcal{E})$ and $f(0) = \mathcal{A}$ and $g(0) = \mathcal{C}$ and $h(0) = \mathcal{D}$ and for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$, and
(ii) there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that $\mathcal{B} = f(\mathcal{E})$ and $A_2 = g(\mathcal{E})$ and $f(0) = \mathcal{A}$ and $g(0) = \mathcal{C}$ and $h(0) = \mathcal{D}$ and for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$.
Then $A_1 = A_2$

provided the parameters meet the following condition:

- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

The scheme *CIRCCMB2'sch 19* deals with unsplit non void strict non empty many sorted signatures $\mathcal{A}$, $\mathcal{B}$ with arity held in gates and Boolean denotation held in gates, a Boolean strict circuit $\mathcal{C}$ of $\mathcal{A}$ with denotation held in gates, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a set $\mathcal{D}$, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{E}$, and states that:

There exists a Boolean strict circuit A of $\mathcal{B}$ with denotation held in gates and there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that

(i) $\mathcal{B} = f(\mathcal{E})$,
(ii) $A = g(\mathcal{E})$,
(iii) $f(0) = \mathcal{A}$,
(iv) $g(0) = \mathcal{C}$,
(v) $h(0) = \mathcal{D}$, and
(vi) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$

provided the following conditions are satisfied:

- Let S be an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates, x be a set, and n be a natural number. Then $\mathcal{F}(S, x, n)$ is unsplit, non void, and strict and has arity held in gates and Boolean denotation held in gates,
- There exist many sorted sets f, h indexed by $\mathbb{N}$ such that
  (i) $\mathcal{B} = f(\mathcal{E})$,
  (ii) $f(0) = \mathcal{A}$,
  (iii) $h(0) = \mathcal{D}$, and
  (iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$,
- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$, and
- Let S, S_1 be unsplit non void strict non empty many sorted signatures with arity held in gates and Boolean denotation held in gates, A be a Boolean strict circuit of S with denotation held in gates, x be a set, and n be a natural number. Suppose $S_1 = \mathcal{F}(S, x, n)$. Then $\mathcal{G}(S, A, x, n)$ is a Boolean strict circuit of S_1 with denotation held in gates.

Let S be a non empty many sorted signature and let A be a set. Let us assume that A is a non-empty algebra over S. The functor $\mathrm{MSAlg}(A, S)$ yielding a non-empty algebra over S is defined as follows:

(Def. 1) $\mathrm{MSAlg}(A, S) = A$.

Now we present two schemes. The scheme *CIRCCMB2'sch 20* deals with unsplit non void strict non empty many sorted signatures $\mathcal{A}$, $\mathcal{B}$ with arity held in gates and Boolean denotation held in gates, a Boolean strict circuit $\mathcal{C}$ of $\mathcal{A}$ with denotation held in gates, a binary functor $\mathcal{F}$ yielding an unsplit non void non empty many sorted signature with arity held in gates and Boolean denotation held in gates, a binary functor $\mathcal{G}$ yielding a set, a set $\mathcal{D}$, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{E}$, and states that:

> There exists a Boolean strict circuit A of $\mathcal{B}$ with denotation held in gates and there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that
>
> (i) $\mathcal{B} = f(\mathcal{E})$,
>
> (ii) $A = g(\mathcal{E})$,
>
> (iii) $f(0) = \mathcal{A}$,
>
> (iv) $g(0) = \mathcal{C}$,
>
> (v) $h(0) = \mathcal{D}$, and
>
> (vi) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A_1 over S and for every set x and for every non-empty algebra A_2 over $\mathcal{F}(x, n)$ such that $S = f(n)$ and $A_1 = g(n)$ and $x = h(n)$ and $A_2 = \mathcal{G}(x, n)$ holds $f(n+1) = S + \cdot \mathcal{F}(x, n)$ and $g(n+1) = A_1 + \cdot A_2$ and $h(n+1) = \mathcal{H}(x, n)$

provided the parameters meet the following requirements:

- There exist many sorted sets f, h indexed by $\mathbb{N}$ such that

  (i) $\mathcal{B} = f(\mathcal{E})$,

  (ii) $f(0) = \mathcal{A}$,

  (iii) $h(0) = \mathcal{D}$, and

  (iv) for every natural number n and for every non empty many sorted signature S and for every set x such that $S = f(n)$ and $x = h(n)$ holds $f(n+1) = S + \cdot \mathcal{F}(x, n)$ and $h(n+1) = \mathcal{H}(x, n)$,

  and
- Let x be a set and n be a natural number. Then $\mathcal{G}(x, n)$ is a Boolean strict circuit of $\mathcal{F}(x, n)$ with denotation held in gates.

The scheme *CIRCCMB2'sch 21* deals with a non empty many sorted signature $\mathcal{A}$, an unsplit non void strict non empty many sorted signature $\mathcal{B}$ with arity held in gates and Boolean denotation held in gates, a non-empty algebra $\mathcal{C}$ over $\mathcal{A}$, a set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding a non empty many sorted signature, a 4-ary functor $\mathcal{G}$ yielding a set, a binary functor $\mathcal{H}$ yielding a set, and a natural number $\mathcal{E}$, and states that:

Let A_1, A_2 be Boolean strict circuits of $\mathcal{B}$ with denotation held in gates. Suppose that

(i) there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that $\mathcal{B} = f(\mathcal{E})$ and $A_1 = g(\mathcal{E})$ and $f(0) = \mathcal{A}$ and $g(0) = \mathcal{C}$ and $h(0) = \mathcal{D}$ and for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$, and

(ii) there exist many sorted sets f, g, h indexed by $\mathbb{N}$ such that $\mathcal{B} = f(\mathcal{E})$ and $A_2 = g(\mathcal{E})$ and $f(0) = \mathcal{A}$ and $g(0) = \mathcal{C}$ and $h(0) = \mathcal{D}$ and for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A over S and for every set x such that $S = f(n)$ and $A = g(n)$ and $x = h(n)$ holds $f(n+1) = \mathcal{F}(S, x, n)$ and $g(n+1) = \mathcal{G}(S, A, x, n)$ and $h(n+1) = \mathcal{H}(x, n)$.

Then $A_1 = A_2$

provided the parameters have the following property:

- Let S be a non empty many sorted signature, A be a non-empty algebra over S, x be a set, and n be a natural number. Then $\mathcal{G}(S, A, x, n)$ is a non-empty algebra over $\mathcal{F}(S, x, n)$.

5. Stability of Multi Cell Circuit

One can prove the following propositions:

(10) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InnerVertices(S_1) misses InputVertices(S_2) and $S = S_1 +\cdot S_2$. Let C_1 be a non-empty circuit of S_1, C_2 be a non-empty circuit of S_2, and C be a non-empty circuit of S. Suppose $C_1 \approx C_2$ and $C = C_1 +\cdot C_2$. Let s_2 be a state of C_2 and s be a state of C. If $s_2 = s{\restriction}$the carrier of S_2, then Following(s_2) = Following(s)$\restriction$the carrier of S_2.

(11) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 +\cdot S_2$. Let C_1 be a non-empty circuit of S_1, C_2 be a non-empty circuit of S_2, and C be a non-empty circuit of S. Suppose $C_1 \approx C_2$ and $C = C_1 +\cdot C_2$. Let s_1 be a state of C_1 and s be a state of C. If $s_1 = s{\restriction}$the carrier of S_1, then Following(s_1) = Following(s)$\restriction$the carrier of S_1.

(12) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose $S_1 \approx S_2$ and InnerVertices(S_1) misses InputVertices(S_2) and $S = S_1 +\cdot S_2$. Let C_1 be a non-empty circuit of S_1, C_2 be a non-empty circuit of S_2, and C be a non-empty circuit of S. Suppose $C_1 \approx C_2$ and $C = C_1 +\cdot C_2$.

Let s_1 be a state of C_1, s_2 be a state of C_2, and s be a state of C. Suppose $s_1 = s\restriction$the carrier of S_1 and $s_2 = s\restriction$the carrier of S_2 and s_1 is stable and s_2 is stable. Then s is stable.

(13) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose $S_1 \approx S_2$ and InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let C_1 be a non-empty circuit of S_1, C_2 be a non-empty circuit of S_2, and C be a non-empty circuit of S. Suppose $C_1 \approx C_2$ and $C = C_1 {+\cdot} C_2$. Let s_1 be a state of C_1, s_2 be a state of C_2, and s be a state of C. Suppose $s_1 = s\restriction$the carrier of S_1 and $s_2 = s\restriction$the carrier of S_2 and s_1 is stable and s_2 is stable. Then s is stable.

(14) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1. Let n be a natural number. Then Following$(s, n)\restriction$the carrier of $S_1 =$ Following(s_1, n).

(15) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_2) misses InnerVertices(S_1) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_2 be a state of A_2. Suppose $s_2 = s\restriction$the carrier of S_2. Let n be a natural number. Then Following$(s, n)\restriction$the carrier of $S_2 =$ Following(s_2, n).

(16) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stable. Let s_2 be a state of A_2. If $s_2 = s\restriction$the carrier of S_2, then Following$(s)\restriction$the carrier of $S_2 =$ Following(s_2).

(17) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stable. Let s_2 be a state of A_2. If $s_2 = s\restriction$the carrier of S_2 and s_2 is stable, then s is stable.

(18) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A. Suppose s is stable. Then

(i) for every state s_1 of A_1 such that $s_1 = s\restriction$the carrier of S_1 holds s_1 is stable, and

(ii) for every state s_2 of A_2 such that $s_2 = s\restriction$the carrier of S_2 holds s_2 is stable.

(19) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s_1 be a state of A_1, s_2 be a state of A_2, and s be a state of A. Suppose $s_1 = s\restriction$the carrier of S_1 and $s_2 = s\restriction$the carrier of S_2 and s_1 is stable. Let n be a natural number. Then Following$(s, n)\restriction$the carrier of S_2 = Following(s_2, n).

(20) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let n_1, n_2 be natural numbers, s be a state of A, s_1 be a state of A_1, and s_2 be a state of A_2. Suppose $s_1 = s\restriction$the carrier of S_1 and Following(s_1, n_1) is stable and s_2 = Following$(s, n_1)\restriction$the carrier of S_2 and Following(s_2, n_2) is stable. Then Following$(s, n_1 + n_2)$ is stable.

(21) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let n_1, n_2 be natural numbers. Suppose for every state s of A_1 holds Following(s, n_1) is stable and for every state s of A_2 holds Following(s, n_2) is stable. Let s be a state of A. Then Following$(s, n_1 + n_2)$ is stable.

(22) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1. Let s_2 be a state of A_2. Suppose $s_2 = s\restriction$the carrier of S_2. Let n be a natural number. Then Following(s, n) = Following$(s_1, n) {+\cdot}$ Following(s_2, n).

(23) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let n_1, n_2 be natural numbers, s be a state of A, and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1. Let s_2 be a state of A_2. Suppose $s_2 = s\restriction$the carrier

of S_2 and Following(s_1, n_1) is stable and Following(s_2, n_2) is stable. Then Following$(s, \max(n_1, n_2))$ is stable.

(24) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let n be a natural number, s be a state of A, and s_1 be a state of A_1. Suppose $s_1 = s \restriction$the carrier of S_1. Let s_2 be a state of A_2. Suppose $s_2 = s \restriction$the carrier of S_2 but Following(s_1, n) is not stable or Following(s_2, n) is not stable. Then Following(s, n) is not stable.

(25) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let n_1, n_2 be natural numbers. Suppose for every state s of A_1 holds Following(s, n_1) is stable and for every state s of A_2 holds Following(s, n_2) is stable. Let s be a state of A. Then Following$(s, \max(n_1, n_2))$ is stable.

The scheme *CIRCCMB2'sch 22* deals with unsplit non void strict non empty many sorted signatures $\mathcal{A}$, $\mathcal{B}$ with arity held in gates and Boolean denotation held in gates, a Boolean strict circuit $\mathcal{C}$ of $\mathcal{A}$ with denotation held in gates, a Boolean strict circuit $\mathcal{D}$ of $\mathcal{B}$ with denotation held in gates, a binary functor $\mathcal{F}$ yielding an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates, a binary functor $\mathcal{G}$ yielding a set, a many sorted set $\mathcal{E}$ indexed by $\mathbb{N}$, a set $\mathcal{F}$, a binary functor $\mathcal{H}$ yielding a set, and a unary functor $\mathcal{I}$ yielding a natural number, and states that:

For every state s of $\mathcal{D}$ holds Following$(s, \mathcal{I}(0) + \mathcal{I}(2) \cdot \mathcal{I}(1))$ is stable

provided the following conditions are satisfied:

- Let x be a set and n be a natural number. Then $\mathcal{G}(x, n)$ is a Boolean strict circuit of $\mathcal{F}(x, n)$ with denotation held in gates,
- For every state s of $\mathcal{C}$ holds Following$(s, \mathcal{I}(0))$ is stable,
- Let n be a natural number, x be a set, and A be a non-empty circuit of $\mathcal{F}(x, n)$. If $x = \mathcal{E}(n)$ and $A = \mathcal{G}(x, n)$, then for every state s of A holds Following$(s, \mathcal{I}(1))$ is stable,
- There exist many sorted sets f, g indexed by $\mathbb{N}$ such that
  (i) $\mathcal{B} = f(\mathcal{I}(2))$,
  (ii) $\mathcal{D} = g(\mathcal{I}(2))$,
  (iii) $f(0) = \mathcal{A}$,
  (iv) $g(0) = \mathcal{C}$,

(v) $\mathcal{E}(0) = \mathcal{F}$, and

(vi) for every natural number n and for every non empty many sorted signature S and for every non-empty algebra A_1 over S and for every set x and for every non-empty algebra A_2 over $\mathcal{F}(x,n)$ such that $S = f(n)$ and $A_1 = g(n)$ and $x = \mathcal{E}(n)$ and $A_2 = \mathcal{G}(x,n)$ holds $f(n+1) = S{+\cdot}\mathcal{F}(x,n)$ and $g(n+1) = A_1{+\cdot}A_2$ and $\mathcal{E}(n+1) = \mathcal{H}(x,n)$,

- InnerVertices($\mathcal{A}$) is a binary relation and InputVertices($\mathcal{A}$) has no pairs,
- $\mathcal{E}(0) = \mathcal{F}$ and $\mathcal{F} \in$ InnerVertices($\mathcal{A}$),
- For every natural number n and for every set x holds InnerVertices($\mathcal{F}(x,n)$) is a binary relation,
- For every natural number n and for every set x such that $x = \mathcal{E}(n)$ holds InputVertices($\mathcal{F}(x,n)$) $\setminus \{x\}$ has no pairs, and
- For every natural number n and for every set x such that $x = \mathcal{E}(n)$ holds $\mathcal{E}(n+1) = \mathcal{H}(x,n)$ and $x \in$ InputVertices($\mathcal{F}(x,n)$) and $\mathcal{H}(x,n) \in$ InnerVertices($\mathcal{F}(x,n)$).

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