FORMALIZED MATHEMATICS
Volume 10, Number 1, 2002
University of Białystok

Half Open Intervals in Real Numbers

Yatsuka Nakamura
Shinshu University
Nagano

Summary. Left and right half open intervals in the real line are defined. Their properties are investigated. A class of all finite union of such intervals are, in a sense, closed by operations of union, intersection and the difference of sets.

MML Identifier: RCOMP_2.

The terminology and notation used here are introduced in the following articles: [5], [1], [3], [4], and [2].

In this paper s, g, h, r, p, p_1, p_2, q, q_1, q_2, x, y, z denote real numbers.

The following two propositions are true:

(1) $x < y$ and $x < z$ iff $x < \min(y, z)$.

(2) $y < x$ and $z < x$ iff $\max(y, z) < x$.

Let g, s be real numbers. The functor $[g, s[$ yielding a subset of $\mathbb{R}$ is defined as follows:

(Def. 1) $[g, s[= \{r; r$ ranges over real numbers: $g \leqslant r \ \wedge \ r < s\}$.

The functor $]g, s]$ yields a subset of $\mathbb{R}$ and is defined as follows:

(Def. 2) $]g, s] = \{r; r$ ranges over real numbers: $g < r \ \wedge \ r \leqslant s\}$.

Next we state a number of propositions:

(3) $r \in [p, q[$ iff $p \leqslant r$ and $r < q$.

(4) $r \in]p, q]$ iff $p < r$ and $r \leqslant q$.

(5) For all g, s such that $g < s$ holds $[g, s[=]g, s[\cup \{g\}$.

(6) For all g, s such that $g < s$ holds $]g, s] =]g, s[\cup \{s\}$.

(7) $[g, g[= \emptyset$.

(8) $]g, g] = \emptyset$.

(9) If $p \leqslant g$, then $[g, p[= \emptyset$.

ISSN 1426–2630

(10) If $p \leqslant g$, then $]g,p] = \emptyset$.

(11) If $g \leqslant p$ and $p \leqslant h$, then $[g,p[\cup [p,h[= [g,h[$.

(12) If $g \leqslant p$ and $p \leqslant h$, then $]g,p] \cup]p,h] =]g,h]$.

(13) If $g \leqslant p_1$ and $g \leqslant p_2$ and $p_1 \leqslant h$ and $p_2 \leqslant h$, then $[g,h] = [g,p_1[\cup [p_1,p_2] \cup]p_2,h]$.

(14) If $g < p_1$ and $g < p_2$ and $p_1 < h$ and $p_2 < h$, then $]g,h[=]g,p_1] \cup]p_1,p_2[\cup [p_2,h[$.

(15) $[q_1,q_2[\cap [p_1,p_2[= [\max(q_1,p_1), \min(q_2,p_2)[$.

(16) $]q_1,q_2] \cap]p_1,p_2] =]\max(q_1,p_1), \min(q_2,p_2)]$.

(17) $]p,q[\subseteq [p,q[$ and $]p,q[\subseteq]p,q]$ and $[p,q[\subseteq [p,q]$ and $]p,q] \subseteq [p,q]$.

(18) If $r \in [p,g[$ and $s \in [p,g[$, then $[r,s] \subseteq [p,g[$.

(19) If $r \in]p,g]$ and $s \in]p,g]$, then $[r,s] \subseteq]p,g]$.

(20) If $p \leqslant q$ and $q \leqslant r$, then $[p,q] \cup]q,r] = [p,r]$.

(21) If $p \leqslant q$ and $q \leqslant r$, then $[p,q[\cup [q,r] = [p,r]$.

(22) If $[q_1,q_2[$ meets $[p_1,p_2[$, then $q_2 \geqslant p_1$.

(23) If $]q_1,q_2]$ meets $]p_1,p_2]$, then $q_2 \geqslant p_1$.

(24) If $[q_1,q_2[$ meets $[p_1,p_2[$, then $[q_1,q_2[\cup [p_1,p_2[= [\min(q_1,p_1), \max(q_2,p_2)[$.

(25) If $]q_1,q_2]$ meets $]p_1,p_2]$, then $]q_1,q_2] \cup]p_1,p_2] =]\min(q_1,p_1), \max(q_2,p_2)]$.

(26) If $[p_1,p_2[$ meets $[q_1,q_2[$, then $[p_1,p_2[\setminus [q_1,q_2[= [p_1,q_1[\cup [q_2,p_2[$.

(27) If $]p_1,p_2]$ meets $]q_1,q_2]$, then $]p_1,p_2] \setminus]q_1,q_2] =]p_1,q_1] \cup]q_2,p_2]$.

References

[1] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.

[2] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[3] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.

[4] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[5] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

Received February 1, 2002