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Preparing the Internal Approximations of Simple Closed Curves[1]

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Summary. We mean by an internal approximation of a simple closed curve a special polygon disjoint with it but sufficiently close to it, i.e. such that it is clock-wise oriented and its right cells meet the curve. We prove lemmas used in the next article to construct a sequence of internal approximations.

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The articles [18], [5], [20], [11], [1], [16], [2], [21], [4], [3], [12], [17], [7], [8], [9], [10], [13], [14], [15], [6], and [19] provide the terminology and notation for this paper.

In this paper j, k, n are natural numbers and C is a subset of $\mathcal{E}^2_{\rm T}$ satisfying conditions of simple closed curve.

Let us consider C. The functor ApproxIndex C yielding a natural number is defined by:

(Def. 1) ApproxIndex C is sufficiently large for C and for every j such that j is sufficiently large for C holds $j \geqslant$ ApproxIndex C.

Next we state the proposition

(1) ApproxIndex $C \geqslant 1$.

Let us consider C. The functor Y-InitStart C yields a natural number and is defined as follows:

(Def. 2) Y-InitStart $C <$ width Gauge(C, ApproxIndex C) and cell(Gauge(C, ApproxIndex C), X-SpanStart(C, ApproxIndex C) $-'$ 1, Y-InitStart C) $\subseteq$ BDD C and for every j such that $j <$ width Gauge(C, ApproxIndex C) and cell(Gauge(C, ApproxIndex C), X-SpanStart(C, ApproxIndex C) $-'$ 1, j) $\subseteq$ BDD C holds $j \geqslant$ Y-InitStart C.

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The following propositions are true:

(2) Y-InitStart $C > 1$.

(3) Y-InitStart $C + 1 <$ width Gauge(C, ApproxIndex C).

Let us consider C, n. Let us assume that n is sufficiently large for C. The functor Y-SpanStart(C, n) yields a natural number and is defined by the conditions (Def. 3).

(Def. 3)(i) Y-SpanStart$(C, n) \leqslant$ width Gauge(C, n),

(ii) for every k such that Y-SpanStart$(C, n) \leqslant k$ and $k \leqslant 2^{n -' \mathrm{ApproxIndex}\, C} \cdot$ (Y-InitStart $C -' 2) + 2$ holds cell(Gauge(C, n), X-SpanStart$(C, n) -' 1, k) \subseteq$ BDD C, and

(iii) for every j such that $j \leqslant$ width Gauge(C, n) and for every k such that $j \leqslant k$ and $k \leqslant 2^{n -' \mathrm{ApproxIndex}\, C} \cdot$ (Y-InitStart $C -' 2) + 2$ holds cell(Gauge(C, n), X-SpanStart$(C, n) -' 1, k) \subseteq$ BDD C holds $j \geqslant$ Y-SpanStart(C, n).

One can prove the following propositions:

(4) If n is sufficiently large for C, then X-SpanStart$(C, n) = 2^{n -' \mathrm{ApproxIndex}\, C} \cdot$ (X-SpanStart(C, ApproxIndex C) $- 2) + 2$.

(5) If n is sufficiently large for C, then Y-SpanStart$(C, n) \leqslant 2^{n -' \mathrm{ApproxIndex}\, C} \cdot$ (Y-InitStart $C -' 2) + 2$.

(6) If n is sufficiently large for C, then cell(Gauge(C, n), X-SpanStart$(C, n) -' 1$, Y-SpanStart$(C, n)) \subseteq$ BDD C.

(7) If n is sufficiently large for C, then $1 <$ Y-SpanStart(C, n) and Y-SpanStart$(C, n) \leqslant$ width Gauge(C, n).

(8) If n is sufficiently large for C, then $\langle$X-SpanStart(C, n), Y-SpanStart$(C, n)\rangle \in$ the indices of Gauge(C, n).

(9) If n is sufficiently large for C, then $\langle$X-SpanStart$(C, n) -' 1$, Y-SpanStart$(C, n)\rangle \in$ the indices of Gauge(C, n).

(10) If n is sufficiently large for C, then cell(Gauge(C, n), X-SpanStart$(C, n) -' 1$, Y-SpanStart$(C, n) -' 1)$ meets C.

(11) If n is sufficiently large for C, then cell(Gauge(C, n), X-SpanStart$(C, n) -' 1$, Y-SpanStart$(C, n))$ misses C.

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