FORMALIZED MATHEMATICS
Volume 10, Number 2, 2002
University of Białystok

Properties of the Internal Approximation of Jordan's Curve[1]

Robert Milewski
University of Białystok

MML Identifier: JORDAN14.

The articles [19], [25], [14], [10], [1], [16], [2], [3], [24], [11], [18], [9], [26], [6], [17], [7], [8], [12], [13], [20], [15], [4], [5], [21], [23], and [22] provide the notation and terminology for this paper.

One can prove the following propositions:

(1) For every non constant standard special circular sequence f holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(2) For every non constant standard special circular sequence f holds $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(3) Let G be a Go-board, f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and k be a natural number. Suppose $1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then $\mathrm{left\_cell}(f, k, G)$ is closed.

(4) Let G be a Go-board, p be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose $1 \leqslant i$ and $i + 1 \leqslant \mathrm{len}\, G$ and $1 \leqslant j$ and $j + 1 \leqslant \mathrm{width}\, G$. Then $p \in \mathrm{Int}\,\mathrm{cell}(G, i, j)$ if and only if the following conditions are satisfied:

(i) $(G \circ (i, j))_{\mathbf{1}} < p_{\mathbf{1}}$,

(ii) $p_{\mathbf{1}} < (G \circ (i + 1, j))_{\mathbf{1}}$,

(iii) $(G \circ (i, j))_{\mathbf{2}} < p_{\mathbf{2}}$, and

(iv) $p_{\mathbf{2}} < (G \circ (i, j + 1))_{\mathbf{2}}$.

(5) For every non constant standard special circular sequence f holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

Let f be a non constant standard special circular sequence. Observe that $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

[1]This work has been partially supported by CALCULEMUS grant HPRN-CT-2000-00102.

ISSN 1426–2630

Let C be a simple closed curve and let n be a natural number. The functor $\mathrm{SpanStart}(C, n)$ yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 1) $\mathrm{SpanStart}(C, n) = \mathrm{Gauge}(C, n)\circ(\text{X-SpanStart}(C, n), \text{Y-SpanStart}(C, n))$.

The following four propositions are true:

(6) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $(\mathrm{Span}(C, n))_1 = \mathrm{SpanStart}(C, n)$.

(7) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{SpanStart}(C, n) \in \mathrm{BDD}\, C$.

(8) Let C be a simple closed curve and n, k be natural numbers. Suppose n is sufficiently large for C. Suppose $1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, \mathrm{Span}(C, n)$. Then $\mathrm{right\_cell}(\mathrm{Span}(C, n), k, \mathrm{Gauge}(C, n))$ misses C and $\mathrm{left\_cell}(\mathrm{Span}(C, n), k, \mathrm{Gauge}(C, n))$ meets C.

(9) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C, n))$.

Let C be a simple closed curve and let n be a natural number. Observe that $\overline{\mathrm{RightComp}(\mathrm{Span}(C, n))}$ is compact.

Next we state a number of propositions:

(10) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C meets $\mathrm{LeftComp}(\mathrm{Span}(C, n))$.

(11) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C misses $\mathrm{RightComp}(\mathrm{Span}(C, n))$.

(12) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C, n))$.

(13) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $C \subseteq \mathrm{UBD}\, \widetilde{\mathcal{L}}(\mathrm{Span}(C, n))$.

(14) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{BDD}\, \widetilde{\mathcal{L}}(\mathrm{Span}(C, n)) \subseteq \mathrm{BDD}\, C$.

(15) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{UBD}\, C \subseteq \mathrm{UBD}\, \widetilde{\mathcal{L}}(\mathrm{Span}(C, n))$.

(16) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{RightComp}(\mathrm{Span}(C, n)) \subseteq \mathrm{BDD}\, C$.

(17) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{UBD}\, C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C, n))$.

(18) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\, C$ misses $\mathrm{BDD}\, \widetilde{\mathcal{L}}(\mathrm{Span}(C, n))$.

(19) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\, C$ misses $\mathrm{RightComp}(\mathrm{Span}(C, n))$.

(20) Let C be a simple closed curve, P be a subset of $\mathcal{E}^2_{\mathrm{T}}$, and n be a natural number. Suppose n is sufficiently large for C. If P is outside component

of C, then P misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(21) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\,C$ misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(22) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(23) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leqslant k$ and $k \leqslant \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i > 1$.

(24) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leqslant k$ and $k \leqslant \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$.

(25) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leqslant k$ and $k \leqslant \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $j > 1$.

(26) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leqslant k$ and $k \leqslant \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $j < \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(27) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\text{Y-SpanStart}(C,n) < \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(28) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m be natural numbers. If $m \geqslant n$ and $n \geqslant 1$, then $\text{X-SpanStart}(C,m) = 2^{m -' n} \cdot (\text{X-SpanStart}(C,n) - 2) + 2$.

(29) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m be natural numbers. Suppose $n \leqslant m$ and n is sufficiently large for C. Then m is sufficiently large for C.

(30) Let G be a Go-board, f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose f is a sequence which elements belong to G and special and $i \leqslant \mathrm{len}\,G$ and $j \leqslant \mathrm{width}\,G$. Then $\mathrm{cell}(G,i,j) \setminus \widetilde{\mathcal{L}}(f)$ is connected.

(31) Let C be a simple closed curve and n, k be natural numbers. Suppose n is sufficiently large for C and $\text{Y-SpanStart}(C,n) \leqslant k$ and $k \leqslant 2^{n -' \mathrm{ApproxIndex}\,C} \cdot (\text{Y-InitStart}\,C -' 2) + 2$. Then $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, k) \setminus \widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq$
$\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(32) Let C be a subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m, i be natural numbers. If $m \leqslant n$ and $1 < i$ and $i + 1 < \mathrm{len}\,\mathrm{Gauge}(C,m)$, then $2^{n -' m} \cdot (i - 2) + 2 + 1 <$

len Gauge(C, n).

(33) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leqslant m$, then RightComp(Span(C, n)) meets RightComp(Span(C, m)).

(34) Let G be a Go-board and f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose f is a sequence which elements belong to G and special. Let i, j be natural numbers. If $i \leqslant \operatorname{len} G$ and $j \leqslant \operatorname{width} G$, then Int cell$(G, i, j) \subseteq (\widetilde{\mathcal{L}}(f))^{\rm c}$.

(35) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leqslant m$, then $\widetilde{\mathcal{L}}$(Span(C, m)) $\subseteq$ $\overline{\text{LeftComp(Span}(C, n))}$.

(36) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leqslant m$, then RightComp(Span(C, n)) $\subseteq$ RightComp(Span(C, m)).

(37) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leqslant m$, then LeftComp(Span(C, m)) $\subseteq$ LeftComp(Span(C, n)).

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek. Countable sets and Hessenberg's theorem. *Formalized Mathematics*, 2(**1**):65–69, 1991.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Czesław Byliński. Gauges. *Formalized Mathematics*, 8(**1**):25–27, 1999.

[5] Czesław Byliński. Some properties of cells on Go-board. *Formalized Mathematics*, 8(**1**):139–146, 1999.

[6] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.

[7] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.

[8] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[9] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Simple closed curves. *Formalized Mathematics*, 2(**5**):663–664, 1991.

[10] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[11] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.

[13] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons. Part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.

[14] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.

[15] Yatsuka Nakamura, Andrzej Trybulec, and Czesław Byliński. Bounded domains and unbounded domains. *Formalized Mathematics*, 8(**1**):1–13, 1999.

[16] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[17] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.

[18] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[19] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[20] Andrzej Trybulec. Left and right component of the complement of a special closed curve. *Formalized Mathematics*, 5(**4**):465–468, 1996.
[21] Andrzej Trybulec. More on the external approximation of a continuum. *Formalized Mathematics*, 9(**4**):831–841, 2001.
[22] Andrzej Trybulec. Introducing spans. *Formalized Mathematics*, 10(**2**):97–98, 2002.
[23] Andrzej Trybulec. Preparing the internal approximations of simple closed curves. *Formalized Mathematics*, 10(**2**):85–87, 2002.
[24] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[25] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[26] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received June 27, 2002