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The Class of Series-Parallel Graphs. Part I

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Summary. The paper introduces some preliminary notions concerning the graph theory according to [20]. We define Necklace n as a graph with vertex $\{1, 2, 3, \ldots, n\}$ and edge set $\{(1, 2), (2, 3), \ldots, (n-1, n)\}$. The goal of the article is to prove that Necklace n and Complement of Necklace n are isomorphic for $n = 0, 1, 4$.

MML Identifier: NECKLACE.

The terminology and notation used in this paper are introduced in the following papers: [23], [22], [25], [12], [1], [15], [5], [11], [2], [24], [26], [28], [18], [6], [7], [21], [13], [19], [27], [8], [9], [10], [17], [3], [4], [14], and [16].

1. Preliminaries

We adopt the following rules: n is a natural number and x_1, x_2, x_3, x_4, x_5, y_1, y_2, y_3 are sets.

Let x_1, x_2, x_3, x_4, x_5 be sets. We say that x_1, x_2, x_3, x_4, x_5 are mutually different if and only if:

(Def. 1) $x_1 \neq x_2$ and $x_1 \neq x_3$ and $x_1 \neq x_4$ and $x_1 \neq x_5$ and $x_2 \neq x_3$ and $x_2 \neq x_4$ and $x_2 \neq x_5$ and $x_3 \neq x_4$ and $x_3 \neq x_5$ and $x_4 \neq x_5$.

Next we state several propositions:

(1) If x_1, x_2, x_3, x_4, x_5 are mutually different, then $\mathrm{card}\{x_1, x_2, x_3, x_4, x_5\} = 5$.

(2) $4 = \{0, 1, 2, 3\}$.

(3) $[\!:\{x_1, x_2, x_3\}, \{y_1, y_2, y_3\}:\!] = \{\langle x_1, y_1\rangle, \langle x_1, y_2\rangle, \langle x_1, y_3\rangle, \langle x_2, y_1\rangle, \langle x_2, y_2\rangle, \langle x_2, y_3\rangle, \langle x_3, y_1\rangle, \langle x_3, y_2\rangle, \langle x_3, y_3\rangle\}$.

(4) For every set x and for every natural number n such that $x \in n$ holds x is a natural number.

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(5) For every non empty natural number x holds $0 \in x$.

Let us observe that every set which is natural is also cardinal.

Let X be a set. One can check that δ_X is one-to-one.

Next we state the proposition

(6) For every set X holds $\overline{\overline{\triangle_X}} = \overline{\overline{X}}$.

Let R be a trivial binary relation. Observe that $\mathrm{dom}\, R$ is trivial.

Let us observe that every function which is trivial is also one-to-one.

We now state several propositions:

(7) For all functions f, g such that $\mathrm{dom}\, f$ misses $\mathrm{dom}\, g$ holds $\mathrm{rng}(f{+}{\cdot}g) = \mathrm{rng}\, f \cup \mathrm{rng}\, g$.

(8) For all one-to-one functions f, g such that $\mathrm{dom}\, f$ misses $\mathrm{dom}\, g$ and $\mathrm{rng}\, f$ misses $\mathrm{rng}\, g$ holds $(f{+}{\cdot}g)^{-1} = f^{-1}{+}{\cdot}g^{-1}$.

(9) For all sets A, a, b holds $(A \longmapsto a){+}{\cdot}(A \longmapsto b) = A \longmapsto b$.

(10) For all sets a, b holds $(a \dot{\longmapsto} b)^{-1} = b \dot{\longmapsto} a$.

(11) For all sets a, b, c, d such that $a = b$ iff $c = d$ holds $[a \longmapsto c, b \longmapsto d]^{-1} = [c \longmapsto a, d \longmapsto b]$.

The scheme *Convers* deals with a non empty set $\mathcal{A}$, a binary relation $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and a unary predicate $\mathcal{P}$, and states that:

$$\mathcal{B}^{\smile} = \{\langle \mathcal{F}(x), \mathcal{G}(x)\rangle; x \text{ ranges over elements of } \mathcal{A} : \mathcal{P}[x]\}$$

provided the parameters meet the following condition:

- $\mathcal{B} = \{\langle \mathcal{G}(x), \mathcal{F}(x)\rangle; x \text{ ranges over elements of } \mathcal{A} : \mathcal{P}[x]\}$.

Next we state the proposition

(12) For all natural numbers i, j, n such that $i < j$ and $j \in n$ holds $i \in n$.

2. Auxiliary Concepts

Let R, S be non empty relational structures. We say that S embeds R if and only if the condition (Def. 2) is satisfied.

(Def. 2) There exists a map f from R into S such that

(i) f is one-to-one, and

(ii) for all elements x, y of the carrier of R holds $\langle x, y\rangle \in$ the internal relation of R iff $\langle f(x), f(y)\rangle \in$ the internal relation of S.

Let us note that the predicate S embeds R is reflexive.

One can prove the following proposition

(13) For all non empty relational structures R, S, T such that R embeds S and S embeds T holds R embeds T.

Let R, S be non empty relational structures. We say that R is equimorphic to S if and only if:

(Def. 3) R embeds S and S embeds R.

Let us notice that the predicate R is equimorphic to S is reflexive and symmetric.

The following proposition is true

(14) Let R, S, T be non empty relational structures. Suppose R is equimorphic to S and S is equimorphic to T. Then R is equimorphic to T.

Let R be a non empty relational structure. We introduce R is parallel as an antonym of R is connected.

Let R be a relational structure. We say that R is symmetric if and only if:

(Def. 4) The internal relation of R is symmetric in the carrier of R.

Let R be a relational structure. We say that R is asymmetric if and only if:

(Def. 5) The internal relation of R is asymmetric.

We now state the proposition

(15) Let R be a relational structure. Suppose R is asymmetric. Then the internal relation of R misses (the internal relation of $R)^{\smile}$.

Let R be a relational structure. We say that R is irreflexive if and only if:

(Def. 6) For every set x such that $x \in$ the carrier of R holds $\langle x, x\rangle \notin$ the internal relation of R.

Let n be a natural number. The functor n-SuccRelStr yielding a strict relational structure is defined as follows:

(Def. 7) The carrier of n-SuccRelStr $= n$ and the internal relation of n-SuccRelStr $= \{\langle i, i+1\rangle; i$ ranges over natural numbers: $i+1 < n\}$.

The following propositions are true:

(16) For every natural number n holds n-SuccRelStr is asymmetric.

(17) If $n > 0$, then $\overline{\overline{\text{the internal relation of } n\text{-SuccRelStr}}} = n - 1$.

Let R be a relational structure. The functor SymRelStr R yielding a strict relational structure is defined by the conditions (Def. 8).

(Def. 8)(i) The carrier of SymRelStr $R =$ the carrier of R, and

(ii) the internal relation of SymRelStr $R =$ (the internal relation of $R)\cup$(the internal relation of $R)^{\smile}$.

Let R be a relational structure. Note that SymRelStr R is symmetric.

Let us mention that there exists a relational structure which is non empty and symmetric.

Let R be a symmetric relational structure. One can verify that the internal relation of R is symmetric.

Let R be a relational structure. The functor ComplRelStr R yielding a strict relational structure is defined by the conditions (Def. 9).

(Def. 9)(i) The carrier of ComplRelStr $R =$ the carrier of R, and

(ii) the internal relation of ComplRelStr $R =$ (the internal relation of $R)^{\rm c} \setminus \triangle_{\text{the carrier of } R}$.

Let R be a non empty relational structure. Observe that ComplRelStr R is non empty.

Next we state the proposition

(18) Let S, R be relational structures. Suppose S and R are isomorphic. Then $\overline{\overline{\text{the internal relation of } S}} = \overline{\overline{\text{the internal relation of } R}}$.

3. Necklace n

Let n be a natural number. The functor Necklace n yielding a strict relational structure is defined as follows:

(Def. 10) Necklace n = SymRelStr n-SuccRelStr .

Let n be a natural number. One can check that Necklace n is symmetric.

We now state two propositions:

(19) The internal relation of Necklace $n = \{\langle i, i+1 \rangle; i$ ranges over natural numbers: $i+1 < n\} \cup \{\langle i+1, i \rangle; i$ ranges over natural numbers: $i+1 < n\}$.

(20) Let x be a set. Then $x \in$ the internal relation of Necklace n if and only if there exists a natural number i such that $i+1 < n$ but $x = \langle i, i+1 \rangle$ or $x = \langle i+1, i \rangle$.

Let n be a natural number. Observe that Necklace n is irreflexive.

Next we state the proposition

(21) For every natural number n holds the carrier of Necklace $n = n$.

Let n be a non empty natural number. Observe that Necklace n is non empty.

Let n be a natural number. Observe that the carrier of Necklace n is finite.

One can prove the following propositions:

(22) For all natural numbers n, i such that $i+1 < n$ holds $\langle i, i+1 \rangle \in$ the internal relation of Necklace n.

(23) For every natural number n and for every natural number i such that $i \in$ the carrier of Necklace n holds $i < n$.

(24) For every non empty natural number n holds Necklace n is connected.

(25) For all natural numbers i, j such that $\langle i, j \rangle \in$ the internal relation of Necklace n holds $i = j+1$ or $j = i+1$.

(26) Let i, j be natural numbers. Suppose $i = j+1$ or $j = i+1$ but $i \in$ the carrier of Necklace n but $j \in$ the carrier of Necklace n. Then $\langle i, j \rangle \in$ the internal relation of Necklace n.

(27) If $n > 0$, then $\overline{\overline{\{\langle i+1, i \rangle; i \text{ ranges over natural numbers: } i+1 < n\}}} = n - 1$.

(28) If $n > 0$, then $\overline{\overline{\text{the internal relation of Necklace } n}} = 2 \cdot (n-1)$.

(29) Necklace 1 and ComplRelStr Necklace 1 are isomorphic.

(30) Necklace 4 and ComplRelStr Necklace 4 are isomorphic.

(31) If Necklace n and ComplRelStr Necklace n are isomorphic, then $n = 0$ or $n = 1$ or $n = 4$.

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