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Behaviour of an Arc Crossing a Line

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In two-dimensional Euclidean space, we examine behaviour of an arc when it crosses a vertical line. There are three types when an arc enters into a line, which are: "Left-In", "Right-In" and "Oscillating-In". Also, there are three types when an arc goes out from a line, which are: "Left-Out", "Right-Out" and "Oscillating-Out". If an arc is a special polygonal arc, there are only two types for each case, entering in and going out. They are "Left-In" and "Right-In" for entering in, and "Left-Out" and "Right-Out" for going out.

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The articles [23], [26], [27], [7], [20], [16], [5], [15], [19], [24], [11], [6], [12], [9], [21], [10], [22], [2], [3], [14], [17], [18], [25], [4], [13], [1], and [8] provide the terminology and notation for this paper.

The following propositions are true:

(1) For every subset P of $\mathcal{E}^2_{\rm T}$ and for all points p_1, p_2, p of $\mathcal{E}^2_{\rm T}$ such that P is an arc from p_1 to p_2 and $p \in P$ holds $\mathrm{Segment}(P, p_1, p_2, p, p) = \{p\}$.

(2) For all points p_1, p_2, p of $\mathcal{E}^2_{\rm T}$ and for every real number a such that $p \in \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{1}} \leqslant a$ and $(p_2)_{\mathbf{1}} \leqslant a$ holds $p_{\mathbf{1}} \leqslant a$.

(3) For all points p_1, p_2, p of $\mathcal{E}^2_{\rm T}$ and for every real number a such that $p \in \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{1}} \geqslant a$ and $(p_2)_{\mathbf{1}} \geqslant a$ holds $p_{\mathbf{1}} \geqslant a$.

(4) For all points p_1, p_2, p of $\mathcal{E}^2_{\rm T}$ and for every real number a such that $p \in \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{1}} < a$ and $(p_2)_{\mathbf{1}} < a$ holds $p_{\mathbf{1}} < a$.

(5) For all points p_1, p_2, p of $\mathcal{E}^2_{\rm T}$ and for every real number a such that $p \in \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{1}} > a$ and $(p_2)_{\mathbf{1}} > a$ holds $p_{\mathbf{1}} > a$.

In the sequel j is a natural number.

Next we state two propositions:

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(6) Let f be a S-sequence in $\mathbb{R}^2$ and p, q be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $1 \leqslant j$ and $j < \operatorname{len} f$ and $p \in \mathcal{L}(f, j)$ and $q \in \mathcal{L}(f, j)$ and $(f_j)_{\mathbf{2}} = (f_{j+1})_{\mathbf{2}}$ and $(f_j)_{\mathbf{1}} > (f_{j+1})_{\mathbf{1}}$ and LE p, q, $\widetilde{\mathcal{L}}(f)$, f_1, $f_{\operatorname{len} f}$. Then $p_{\mathbf{1}} \geqslant q_{\mathbf{1}}$.

(7) Let f be a S-sequence in $\mathbb{R}^2$ and p, q be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $1 \leqslant j$ and $j < \operatorname{len} f$ and $p \in \mathcal{L}(f, j)$ and $q \in \mathcal{L}(f, j)$ and $(f_j)_{\mathbf{2}} = (f_{j+1})_{\mathbf{2}}$ and $(f_j)_{\mathbf{1}} < (f_{j+1})_{\mathbf{1}}$ and LE p, q, $\widetilde{\mathcal{L}}(f)$, f_1, $f_{\operatorname{len} f}$. Then $p_{\mathbf{1}} \leqslant q_{\mathbf{1}}$.

Let P be a subset of $\mathcal{E}^2_{\mathrm{T}}$, let p_1, p_2, p be points of $\mathcal{E}^2_{\mathrm{T}}$, and let e be a real number. We say that p is LIn of P, p_1, p_2, e if and only if the conditions (Def. 1) are satisfied.

(Def. 1)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that $(p_4)_{\mathbf{1}} < e$ and LE p_4, p, P, p_1, p_2 and for every point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_4, p_5, P, p_1, p_2 and LE p_5, p, P, p_1, p_2 holds $(p_5)_{\mathbf{1}} \leqslant e$.

We say that p is RIn of P, p_1, p_2, e if and only if the conditions (Def. 2) are satisfied.

(Def. 2)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that $(p_4)_{\mathbf{1}} > e$ and LE p_4, p, P, p_1, p_2 and for every point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_4, p_5, P, p_1, p_2 and LE p_5, p, P, p_1, p_2 holds $(p_5)_{\mathbf{1}} \geqslant e$.

We say that p is LOut of P, p_1, p_2, e if and only if the conditions (Def. 3) are satisfied.

(Def. 3)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that $(p_4)_{\mathbf{1}} < e$ and LE p, p_4, P, p_1, p_2 and for every point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_5, p_4, P, p_1, p_2 and LE p, p_5, P, p_1, p_2 holds $(p_5)_{\mathbf{1}} \leqslant e$.

We say that p is ROut of P, p_1, p_2, e if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that $(p_4)_{\mathbf{1}} > e$ and LE p, p_4, P, p_1, p_2 and for every point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_5, p_4, P, p_1, p_2 and LE p, p_5, P, p_1, p_2 holds $(p_5)_{\mathbf{1}} \geqslant e$.

We say that p is OsIn of P, p_1, p_2, e if and only if the conditions (Def. 5) are satisfied.

(Def. 5)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_7 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_7, p, P, p_1, p_2 and for every point p_8 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_7, p_8, P, p_1, p_2 and LE p_8, p, P, p_1, p_2 holds $(p_8)_{\mathbf{1}} = e$ and for every point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_4, p_7, P, p_1, p_2 and $p_4 \neq p_7$ holds there exists a point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_4, p_5, P, p_1, p_2 and LE p_5, p_7, P, p_1, p_2 and $(p_5)_{\mathbf{1}} > e$ and there exists a point p_6 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_4, p_6, P, p_1, p_2 and LE p_6, p_7, P, p_1, p_2 and $(p_6)_{\mathbf{1}} < e$.

We say that p is OsOut of P, p_1, p_2, e if and only if the conditions (Def. 6) are satisfied.

(Def. 6)(i) P is an arc from p_1 to p_2,

(ii) $p \in P$,

(iii) $p_{\mathbf{1}} = e$, and

(iv) there exists a point p_7 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p, p_7, P, p_1, p_2 and for every point p_8 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_8, p_7, P, p_1, p_2 and LE p, p_8, P, p_1, p_2 holds $(p_8)_{\mathbf{1}} = e$ and for every point p_4 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_7, p_4, P, p_1, p_2 and $p_4 \neq p_7$ holds there exists a point p_5 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_5, p_4, P, p_1, p_2 and LE p_7, p_5, P, p_1, p_2 and $(p_5)_{\mathbf{1}} > e$ and there exists a point p_6 of $\mathcal{E}^2_{\mathrm{T}}$ such that LE p_6, p_4, P, p_1, p_2 and LE p_7, p_6, P, p_1, p_2 and $(p_6)_{\mathbf{1}} < e$.

We now state a number of propositions:

(8) Let P be a subset of $\mathcal{E}^2_{\mathrm{T}}$, p_1, p_2, p be points of $\mathcal{E}^2_{\mathrm{T}}$, and e be a real number. Suppose P is an arc from p_1 to p_2 and $(p_1)_{\mathbf{1}} \leqslant e$ and $(p_2)_{\mathbf{1}} \geqslant e$. Then there exists a point p_3 of $\mathcal{E}^2_{\mathrm{T}}$ such that $p_3 \in P$ and $(p_3)_{\mathbf{1}} = e$.

(9) Let P be a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$, p_1, p_2, p be points of $\mathcal{E}^2_{\mathrm{T}}$, and e be a real number. Suppose P is an arc from p_1 to p_2 and $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and $p \in P$ and $p_{\mathbf{1}} = e$. Then p is LIn of P, p_1, p_2, e, RIn of P, p_1, p_2, e, and OsIn of P, p_1, p_2, e.

(10) Let P be a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$, p_1, p_2, p be points of $\mathcal{E}^2_{\mathrm{T}}$, and e be a real number. Suppose P is an arc from p_1 to p_2 and $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and $p \in P$ and $p_{\mathbf{1}} = e$. Then p is LOut of P, p_1, p_2, e, ROut of P, p_1, p_2, e, and OsOut of P, p_1, p_2, e.

(11) For every subset P of $\mathbb{I}$ and for every real number s such that $P = [0, s[$ holds P is open.

(12) For every subset P of $\mathbb{I}$ and for every real number s such that $P =]s, 1]$ holds P is open.

(13) Let P be a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$, P_1 be a subset of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P$, Q be a subset of $\mathbb{I}$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction P$, and s be a real number. Suppose

$s \leqslant 1$ and $P_1 = \{q_0; q_0$ ranges over points of $\mathcal{E}^2_{\rm T}$: $\bigvee_{s_1 : \text{real number}} (0 \leqslant s_1 \wedge s_1 < s \wedge q_0 = f(s_1))\}$ and $Q = [0, s[$. Then $f^\circ Q = P_1$.

(14) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, P_1 be a subset of $(\mathcal{E}^2_{\rm T})\restriction P$, Q be a subset of $\mathbb{I}$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$, and s be a real number. Suppose $s \geqslant 0$ and $P_1 = \{q_0; q_0$ ranges over points of $\mathcal{E}^2_{\rm T}$: $\bigvee_{s_1 : \text{real number}} (s < s_1 \wedge s_1 \leqslant 1 \wedge q_0 = f(s_1))\}$ and $Q =]s, 1]$. Then $f^\circ Q = P_1$.

(15) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, P_1 be a subset of $(\mathcal{E}^2_{\rm T})\restriction P$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$, and s be a real number. Suppose $s \leqslant 1$ and f is a homeomorphism and $P_1 = \{q_0; q_0$ ranges over points of $\mathcal{E}^2_{\rm T}$: $\bigvee_{s_1 : \text{real number}} (0 \leqslant s_1 \wedge s_1 < s \wedge q_0 = f(s_1))\}$. Then P_1 is open.

(16) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, P_1 be a subset of $(\mathcal{E}^2_{\rm T})\restriction P$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$, and s be a real number. Suppose $s \geqslant 0$ and f is a homeomorphism and $P_1 = \{q_0; q_0$ ranges over points of $\mathcal{E}^2_{\rm T}$: $\bigvee_{s_1 : \text{real number}} (s < s_1 \wedge s_1 \leqslant 1 \wedge q_0 = f(s_1))\}$. Then P_1 is open.

(17) Let T be a non empty topological structure, Q_1, Q_2 be subsets of T, and p_1, p_2 be points of T. Suppose $Q_1 \cap Q_2 = \emptyset$ and $Q_1 \cup Q_2 =$ the carrier of T and $p_1 \in Q_1$ and $p_2 \in Q_2$ and Q_1 is open and Q_2 is open. Then it is not true that there exists a map P from $\mathbb{I}$ into T such that P is continuous and $P(0) = p_1$ and $P(1) = p_2$.

(18) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, Q be a subset of $(\mathcal{E}^2_{\rm T})\restriction P$, and p_1, p_2, q be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and $q \in P$ and $q \neq p_1$ and $q \neq p_2$ and $Q = P \setminus \{q\}$. Then Q is not connected and it is not true that there exists a map R from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P\restriction Q$ such that R is continuous and $R(0) = p_1$ and $R(1) = p_2$.

(19) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $q_2 \in P$. Then LE q_1, q_2, P, p_1, p_2 or LE q_2, q_1, P, p_1, p_2.

(20) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $p_1 \neq q_1$. Then $\mathrm{Segment}(P, p_1, p_2, p_1, q_1)$ is an arc from p_1 to q_1.

(21) Let n be a natural number, p_1, p_2 be points of $\mathcal{E}^n_{\rm T}$, and P, P_1 be non empty subsets of $\mathcal{E}^n_{\rm T}$. If P is an arc from p_1 to p_2 and P_1 is an arc from p_1 to p_2 and $P_1 \subseteq P$, then $P_1 = P$.

(22) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $p_2 \neq q_1$. Then $\mathrm{Segment}(P, p_1, p_2, q_1, p_2)$ is an arc from q_1 to p_2.

(23) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1, q_2, q_3 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and LE q_1, q_2, P, p_1, p_2 and LE q_2, q_3, P, p_1, p_2. Then $\mathrm{Segment}(P, p_1, p_2, q_1, q_2) \cup \mathrm{Segment}(P, p_1, p_2, q_2, q_3) = \mathrm{Segment}(P, p_1, p_2, q_1, q_3)$.

(24) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1, q_2, q_3 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and LE q_1, q_2, P, p_1, p_2 and LE q_2, q_3, P, p_1, p_2. Then $\text{Segment}(P, p_1, p_2, q_1, q_2) \cap \text{Segment}(P, p_1, p_2, q_2, q_3) = \{q_2\}$.

(25) For every non empty subset P of $\mathcal{E}^2_{\rm T}$ and for all points p_1, p_2 of $\mathcal{E}^2_{\rm T}$ such that P is an arc from p_1 to p_2 holds $\text{Segment}(P, p_1, p_2, p_1, p_2) = P$.

(26) Let T be a non empty topological space, w_1, w_2, w_3 be points of T, and h_1, h_2 be maps from $\mathbb{I}$ into T. Suppose h_1 is continuous and $w_1 = h_1(0)$ and $w_2 = h_1(1)$ and h_2 is continuous and $w_2 = h_2(0)$ and $w_3 = h_2(1)$. Then there exists a map h_3 from $\mathbb{I}$ into T such that h_3 is continuous and $w_1 = h_3(0)$ and $w_3 = h_3(1)$.

(27) Let T be a non empty topological space, a, b, c be points of T, G_1 be a path from a to b, and G_2 be a path from b to c. Suppose G_1 is continuous and G_2 is continuous and $G_1(0) = a$ and $G_1(1) = b$ and $G_2(0) = b$ and $G_2(1) = c$. Then $G_1 + G_2$ is continuous and $(G_1 + G_2)(0) = a$ and $(G_1 + G_2)(1) = c$.

(28) Let P, Q_1 be non empty subsets of $\mathcal{E}^2_{\rm T}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is an arc from p_1 to p_2 and Q_1 is an arc from q_1 to q_2 and LE q_1, q_2, P, p_1, p_2 and $Q_1 \subseteq P$. Then $Q_1 = \text{Segment}(P, p_1, p_2, q_1, q_2)$.

(29) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1, q_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and q_1 is LIn of P, p_1, p_2, e and $(q_2)_{\mathbf{1}} = e$ and $\mathcal{L}(q_1, q_2) \subseteq P$ and $p \in \mathcal{L}(q_1, q_2)$. Then p is LIn of P, p_1, p_2, e.

(30) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1, q_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and q_1 is RIn of P, p_1, p_2, e and $(q_2)_{\mathbf{1}} = e$ and $\mathcal{L}(q_1, q_2) \subseteq P$ and $p \in \mathcal{L}(q_1, q_2)$. Then p is RIn of P, p_1, p_2, e.

(31) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1, q_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and q_1 is LOut of P, p_1, p_2, e and $(q_2)_{\mathbf{1}} = e$ and $\mathcal{L}(q_1, q_2) \subseteq P$ and $p \in \mathcal{L}(q_1, q_2)$. Then p is LOut of P, p_1, p_2, e.

(32) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1, q_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and q_1 is ROut of P, p_1, p_2, e and $(q_2)_{\mathbf{1}} = e$ and $\mathcal{L}(q_1, q_2) \subseteq P$ and $p \in \mathcal{L}(q_1, q_2)$. Then p is ROut of P, p_1, p_2, e.

(33) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose P is a special polygonal arc joining p_1 and p_2 and $(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and $p \in P$ and $p_{\mathbf{1}} = e$. Then p is LIn of P, p_1, p_2, e and RIn of P, p_1, p_2, e.

(34) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$, p_1, p_2, p be points of $\mathcal{E}^2_{\rm T}$, and e be a real number. Suppose P is a special polygonal arc joining p_1 and p_2 and

$(p_1)_{\mathbf{1}} < e$ and $(p_2)_{\mathbf{1}} > e$ and $p \in P$ and $p_{\mathbf{1}} = e$. Then p is LOut of P, p_1, p_2, e and ROut of P, p_1, p_2, e.

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