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Continuous Mappings between Finite and One-Dimensional Finite Topological Spaces

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Summary. We showed relations between separateness and inflation operation. We also gave some relations between separateness and connectedness defined before. For two finite topological spaces, we defined a continuous function from one to another. Some topological concepts are preserved by such continuous functions. We gave one-dimensional concrete models of finite topological space.

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The notation and terminology used here are introduced in the following articles: [12], [5], [13], [1], [14], [3], [4], [2], [6], [10], [9], [11], [7], and [8].

Let F_1 be a non empty finite topology space and let A, B be subsets of F_1. We say that A and B are separated if and only if:

(Def. 1) A^b misses B and A misses B^b.

Next we state a number of propositions:

(1) Let F_1 be a filled non empty finite topology space, A be a subset of F_1, and n, m be natural numbers. If $n \leqslant m$, then $\mathrm{Finf}(A,n) \subseteq \mathrm{Finf}(A,m)$.

(2) Let F_1 be a filled non empty finite topology space, A be a subset of F_1, and n, m be natural numbers. If $n \leqslant m$, then $\mathrm{Fcl}(A,n) \subseteq \mathrm{Fcl}(A,m)$.

(3) Let F_1 be a filled non empty finite topology space, A be a subset of F_1, and n, m be natural numbers. If $n \leqslant m$, then $\mathrm{Fdfl}(A,m) \subseteq \mathrm{Fdfl}(A,n)$.

(4) Let F_1 be a filled non empty finite topology space, A be a subset of F_1, and n, m be natural numbers. If $n \leqslant m$, then $\mathrm{Fint}(A,m) \subseteq \mathrm{Fint}(A,n)$.

(5) Let F_1 be a non empty finite topology space and A, B be subsets of F_1. If A and B are separated, then B and A are separated.

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(6) Let F_1 be a filled non empty finite topology space and A, B be subsets of F_1. If A and B are separated, then A misses B.

(7) Let F_1 be a non empty finite topology space and A, B be subsets of F_1. Suppose F_1 is symmetric. Then A and B are separated if and only if A^f misses B and A misses B^f.

(8) Let F_1 be a filled non empty finite topology space and A, B be subsets of F_1. If F_1 is symmetric and A^b misses B, then A misses B^b.

(9) Let F_1 be a filled non empty finite topology space and A, B be subsets of F_1. If F_1 is symmetric and A misses B^b, then A^b misses B.

(10) Let F_1 be a filled non empty finite topology space and A, B be subsets of F_1. Suppose F_1 is symmetric. Then A and B are separated if and only if A^b misses B.

(11) Let F_1 be a filled non empty finite topology space and A, B be subsets of F_1. Suppose F_1 is symmetric. Then A and B are separated if and only if A misses B^b.

(12) Let F_1 be a filled non empty finite topology space and I_1 be a subset of F_1. Suppose F_1 is symmetric. Then I_1 is connected if and only if for all subsets A, B of F_1 such that $I_1 = A \cup B$ and A and B are separated holds $A = I_1$ or $B = I_1$.

(13) Let F_1 be a filled non empty finite topology space and B be a subset of F_1. Suppose F_1 is symmetric. Then B is connected if and only if it is not true that there exists a subset C of F_1 such that $C \neq \emptyset$ and $B \setminus C \neq \emptyset$ and $C \subseteq B$ and C^b misses $B \setminus C$.

Let F_2, F_3 be non empty finite topology spaces, let f be a function from the carrier of F_2 into the carrier of F_3, and let n be a natural number. We say that f is continuous n if and only if:

(Def. 2) For every element x of F_2 and for every element y of F_3 such that $x \in$ the carrier of F_2 and $y = f(x)$ holds $f^\circ U(x, 0) \subseteq U(y, n)$.

Next we state four propositions:

(14) Let F_2 be a non empty finite topology space, F_3 be a filled non empty finite topology space, n be a natural number, and f be a function from the carrier of F_2 into the carrier of F_3. If f is continuous 0, then f is continuous n.

(15) Let F_2 be a non empty finite topology space, F_3 be a filled non empty finite topology space, n_0, n be natural numbers, and f be a function from the carrier of F_2 into the carrier of F_3. If f is continuous n_0 and $n_0 \leqslant n$, then f is continuous n.

(16) Let F_2, F_3 be non empty finite topology spaces, A be a subset of F_2, B be a subset of F_3, and f be a function from the carrier of F_2 into the carrier of F_3. If f is continuous 0 and $B = f^\circ A$, then $f^\circ A^b \subseteq B^b$.

(17) Let F_2, F_3 be non empty finite topology spaces, A be a subset of F_2, B be a subset of F_3, and f be a function from the carrier of F_2 into the carrier of F_3. Suppose A is connected and f is continuous 0 and $B = f^\circ A$. Then B is connected.

Let n be a natural number. The functor $\mathrm{Nbdl1}(n)$ yielding a function from $\mathrm{Seg}\, n$ into $2^{\mathrm{Seg}\, n}$ is defined as follows:

(Def. 3) $\mathrm{dom}\, \mathrm{Nbdl1}(n) = \mathrm{Seg}\, n$ and for every natural number i such that $i \in \mathrm{Seg}\, n$ holds $(\mathrm{Nbdl1}(n))(i) = \{i, \max(i -' 1, 1), \min(i + 1, n)\}$.

Let n be a natural number. Let us assume that $n > 0$. The functor $\mathrm{FTSL1}(n)$ yielding a non empty finite topology space is defined as follows:

(Def. 4) $\mathrm{FTSL1}(n) = \langle \mathrm{Seg}\, n, \mathrm{Nbdl1}(n) \rangle$.

We now state two propositions:

(18) For every natural number n such that $n > 0$ holds $\mathrm{FTSL1}(n)$ is filled.

(19) For every natural number n such that $n > 0$ holds $\mathrm{FTSL1}(n)$ is symmetric.

Let n be a natural number. The functor $\mathrm{Nbdc1}(n)$ yielding a function from $\mathrm{Seg}\, n$ into $2^{\mathrm{Seg}\, n}$ is defined by the conditions (Def. 5).

(Def. 5)(i) $\mathrm{dom}\, \mathrm{Nbdc1}(n) = \mathrm{Seg}\, n$, and

(ii) for every natural number i such that $i \in \mathrm{Seg}\, n$ holds if $1 < i$ and $i < n$, then $(\mathrm{Nbdc1}(n))(i) = \{i, i -' 1, i + 1\}$ and if $i = 1$ and $i < n$, then $(\mathrm{Nbdc1}(n))(i) = \{i, n, i + 1\}$ and if $1 < i$ and $i = n$, then $(\mathrm{Nbdc1}(n))(i) = \{i, i -' 1, 1\}$ and if $i = 1$ and $i = n$, then $(\mathrm{Nbdc1}(n))(i) = \{i\}$.

Let n be a natural number. Let us assume that $n > 0$. The functor $\mathrm{FTSC1}(n)$ yielding a non empty finite topology space is defined as follows:

(Def. 6) $\mathrm{FTSC1}(n) = \langle \mathrm{Seg}\, n, \mathrm{Nbdc1}(n) \rangle$.

We now state two propositions:

(20) For every natural number n such that $n > 0$ holds $\mathrm{FTSC1}(n)$ is filled.

(21) For every natural number n such that $n > 0$ holds $\mathrm{FTSC1}(n)$ is symmetric.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[4] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[5] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.

[6] Hiroshi Imura and Masayoshi Eguchi. Finite topological spaces. *Formalized Mathematics*, 3(**2**):189–193, 1992.

[7] Jarosław Kotowicz. The limit of a real function at infinity. *Formalized Mathematics*, 2(**1**):17–28, 1991.
[8] Jarosław Kotowicz and Yuji Sakai. Properties of partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 3(**2**):279–288, 1992.
[9] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.
[10] Masami Tanaka and Yatsuka Nakamura. Some set series in finite topological spaces. Fundamental concepts for image processing. *Formalized Mathematics*, 12(**2**):125–129, 2004.
[11] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[12] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[13] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[14] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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