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On Some Points of a Simple Closed Curve. Part II

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Summary. In the paper we formalize some lemmas needed by the proof of the Jordan Curve Theorem according to [23]. We show basic properties of the upper and the lower approximations of a simple closed curve (as its compactness and connectedness) and some facts about special points of such approximations.

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The notation and terminology used in this paper are introduced in the following papers: [25], [28], [1], [24], [29], [4], [16], [15], [2], [12], [22], [7], [27], [21], [13], [3], [5], [8], [9], [10], [18], [19], [20], [26], [6], [11], [17], and [14].

1. Properties of the Approximations

In this paper C denotes a simple closed curve and i denotes a natural number.

We now state two propositions:

(1) $(\mathrm{UpperAppr}(C))(i) \subseteq \overline{\mathrm{RightComp}(\mathrm{Cage}(C,0))}$.

(2) $(\mathrm{LowerAppr}(C))(i) \subseteq \overline{\mathrm{RightComp}(\mathrm{Cage}(C,0))}$.

Let C be a simple closed curve. One can verify that $\mathrm{UpperArc}(C)$ is connected and $\mathrm{LowerArc}(C)$ is connected.

We now state two propositions:

(3) $(\mathrm{UpperAppr}(C))(i)$ is compact and connected.

(4) $(\mathrm{LowerAppr}(C))(i)$ is compact and connected.

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Let C be a simple closed curve. Observe that NorthArc(C) is compact and SouthArc(C) is compact.

2. On Special Points of Approximations

One can prove the following propositions:

(5) $\mathrm{W}_{\mathrm{min}}(C) \in \mathrm{NorthArc}(C)$.

(6) $\mathrm{E}_{\mathrm{max}}(C) \in \mathrm{NorthArc}(C)$.

(7) $\mathrm{W}_{\mathrm{min}}(C) \in \mathrm{SouthArc}(C)$.

(8) $\mathrm{E}_{\mathrm{max}}(C) \in \mathrm{SouthArc}(C)$.

(9) $\mathrm{UMP}\, C \in \mathrm{NorthArc}(C)$.

(10) $\mathrm{LMP}\, C \in \mathrm{SouthArc}(C)$.

(11) $\mathrm{NorthArc}(C) \subseteq C$.

(12) $\mathrm{SouthArc}(C) \subseteq C$.

(13) $\mathrm{LMP}\, C \in \mathrm{LowerArc}(C)$ and $\mathrm{UMP}\, C \in \mathrm{UpperArc}(C)$ or $\mathrm{UMP}\, C \in \mathrm{LowerArc}(C)$ and $\mathrm{LMP}\, C \in \mathrm{UpperArc}(C)$.

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