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On the Real Valued Functions[1]

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The terminology and notation used here have been introduced in the following articles: [9], [12], [1], [10], [11], [13], [14], [2], [3], [4], [6], [5], [8], and [7].

Let r be a real number. Observe that $\frac{r}{r}$ is non negative.

Let r be a real number. Observe that $r \cdot r$ is non negative and $r \cdot r^{-1}$ is non negative.

Let r be a non negative real number. One can check that $\sqrt{r}$ is non negative.

Let r be a positive real number. Observe that $\sqrt{r}$ is positive.

We now state the proposition

(1) For every function f and for every set A such that f is one-to-one and $A \subseteq \mathrm{dom}(f^{-1})$ holds $f^{\circ}(f^{-1})^{\circ}A = A$.

Let f be a non-empty function. One can verify that $f^{-1}(\{0\})$ is empty.

Let R be a binary relation. We say that R is positive yielding if and only if:

(Def. 1) For every real number r such that $r \in \mathrm{rng}\, R$ holds $0 < r$.

We say that R is negative yielding if and only if:

(Def. 2) For every real number r such that $r \in \mathrm{rng}\, R$ holds $0 > r$.

We say that R is non-positive yielding if and only if:

(Def. 3) For every real number r such that $r \in \mathrm{rng}\, R$ holds $0 \geq r$.

We say that R is non-negative yielding if and only if:

(Def. 4) For every real number r such that $r \in \mathrm{rng}\, R$ holds $0 \leq r$.

Let X be a set and let r be a positive real number. Observe that $X \longmapsto r$ is positive yielding.

Let X be a set and let r be a negative real number. Note that $X \longmapsto r$ is negative yielding.

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Let X be a set and let r be a non positive real number. Note that $X \longmapsto r$ is non-positive yielding.

Let X be a set and let r be a non negative real number. Observe that $X \longmapsto r$ is non-negative yielding.

Let X be a non empty set. Note that $X \longmapsto 0$ is non non-empty.

Let us observe that every binary relation which is positive yielding is also non-negative yielding and non-empty and every binary relation which is negative yielding is also non-positive yielding and non-empty.

Let X be a set. One can check that there exists a function from X into $\mathbb{R}$ which is negative yielding and there exists a function from X into $\mathbb{R}$ which is positive yielding.

One can check that there exists a function which is non-empty and real-yielding.

We now state two propositions:

(2) For every non-empty real-yielding function f holds $\mathrm{dom}(\frac{1}{f}) = \mathrm{dom}\, f$.

(3) Let X be a non empty set, f be a partial function from X to $\mathbb{R}$, and g be a non-empty partial function from X to $\mathbb{R}$. Then $\mathrm{dom}(\frac{f}{g}) = \mathrm{dom}\, f \cap \mathrm{dom}\, g$.

Let X be a set and let f, g be non-positive yielding partial functions from X to $\mathbb{R}$. Observe that $f + g$ is non-positive yielding.

Let X be a set and let f, g be non-negative yielding partial functions from X to $\mathbb{R}$. Note that $f + g$ is non-negative yielding.

Let X be a set, let f be a positive yielding partial function from X to $\mathbb{R}$, and let g be a non-negative yielding partial function from X to $\mathbb{R}$. Observe that $f + g$ is positive yielding.

Let X be a set, let f be a non-negative yielding partial function from X to $\mathbb{R}$, and let g be a positive yielding partial function from X to $\mathbb{R}$. One can verify that $f + g$ is positive yielding.

Let X be a set, let f be a non-positive yielding partial function from X to $\mathbb{R}$, and let g be a negative yielding partial function from X to $\mathbb{R}$. Note that $f + g$ is negative yielding.

Let X be a set, let f be a negative yielding partial function from X to $\mathbb{R}$, and let g be a non-positive yielding partial function from X to $\mathbb{R}$. Note that $f + g$ is negative yielding.

Let X be a set, let f be a non-negative yielding partial function from X to $\mathbb{R}$, and let g be a non-positive yielding partial function from X to $\mathbb{R}$. Note that $f - g$ is non-negative yielding.

Let X be a set, let f be a non-positive yielding partial function from X to $\mathbb{R}$, and let g be a non-negative yielding partial function from X to $\mathbb{R}$. Observe that $f - g$ is non-positive yielding.

Let X be a set, let f be a positive yielding partial function from X to $\mathbb{R}$, and let g be a non-positive yielding partial function from X to $\mathbb{R}$. One can check that $f - g$ is positive yielding.

Let X be a set, let f be a non-positive yielding partial function from X to $\mathbb{R}$, and let g be a positive yielding partial function from X to $\mathbb{R}$. Observe that $f - g$ is negative yielding.

Let X be a set, let f be a negative yielding partial function from X to $\mathbb{R}$, and let g be a non-negative yielding partial function from X to $\mathbb{R}$. Note that $f - g$ is negative yielding.

Let X be a set, let f be a non-negative yielding partial function from X to $\mathbb{R}$, and let g be a negative yielding partial function from X to $\mathbb{R}$. One can verify that $f - g$ is positive yielding.

Let X be a set and let f, g be non-positive yielding partial functions from X to $\mathbb{R}$. One can verify that $f\,g$ is non-negative yielding.

Let X be a set and let f, g be non-negative yielding partial functions from X to $\mathbb{R}$. Note that $f\,g$ is non-negative yielding.

Let X be a set, let f be a non-positive yielding partial function from X to $\mathbb{R}$, and let g be a non-negative yielding partial function from X to $\mathbb{R}$. One can verify that $f\,g$ is non-positive yielding.

Let X be a set, let f be a non-negative yielding partial function from X to $\mathbb{R}$, and let g be a non-positive yielding partial function from X to $\mathbb{R}$. Observe that $f\,g$ is non-positive yielding.

Let X be a set, let f be a positive yielding partial function from X to $\mathbb{R}$, and let g be a negative yielding partial function from X to $\mathbb{R}$. Note that $f\,g$ is negative yielding.

Let X be a set, let f be a negative yielding partial function from X to $\mathbb{R}$, and let g be a positive yielding partial function from X to $\mathbb{R}$. One can verify that $f\,g$ is negative yielding.

Let X be a set and let f, g be positive yielding partial functions from X to $\mathbb{R}$. One can verify that $f\,g$ is positive yielding.

Let X be a set and let f, g be negative yielding partial functions from X to $\mathbb{R}$. One can check that $f\,g$ is positive yielding.

Let X be a set and let f, g be non-empty partial functions from X to $\mathbb{R}$. Observe that $f\,g$ is non-empty.

Let X be a set and let f be a partial function from X to $\mathbb{R}$. Note that $f\,f$ is non-negative yielding.

Let X be a set, let r be a non positive real number, and let f be a non-positive yielding partial function from X to $\mathbb{R}$. One can verify that $r\,f$ is non-negative yielding.

Let X be a set, let r be a non negative real number, and let f be a non-negative yielding partial function from X to $\mathbb{R}$. Observe that $r\,f$ is non-negative yielding.

Let X be a set, let r be a non positive real number, and let f be a non-negative yielding partial function from X to $\mathbb{R}$. One can verify that $r\,f$ is non-positive yielding.

Let X be a set, let r be a non negative real number, and let f be a non-positive yielding partial function from X to $\mathbb{R}$. One can verify that $r\,f$ is non-positive yielding.

Let X be a set, let r be a positive real number, and let f be a negative yielding partial function from X to $\mathbb{R}$. Note that $r\,f$ is negative yielding.

Let X be a set, let r be a negative real number, and let f be a positive yielding partial function from X to $\mathbb{R}$. One can check that $r\,f$ is negative yielding.

Let X be a set, let r be a positive real number, and let f be a positive yielding partial function from X to $\mathbb{R}$. One can verify that $r\,f$ is positive yielding.

Let X be a set, let r be a negative real number, and let f be a negative yielding partial function from X to $\mathbb{R}$. Note that $r\,f$ is positive yielding.

Let X be a set, let r be a non zero real number, and let f be a non-empty partial function from X to $\mathbb{R}$. Observe that $r\,f$ is non-empty.

Let X be a non empty set and let f, g be non-positive yielding partial functions from X to $\mathbb{R}$. Note that $\frac{f}{g}$ is non-negative yielding.

Let X be a non empty set and let f, g be non-negative yielding partial functions from X to $\mathbb{R}$. Observe that $\frac{f}{g}$ is non-negative yielding.

Let X be a non empty set, let f be a non-positive yielding partial function from X to $\mathbb{R}$, and let g be a non-negative yielding partial function from X to $\mathbb{R}$. Note that $\frac{f}{g}$ is non-positive yielding.

Let X be a non empty set, let f be a non-negative yielding partial function from X to $\mathbb{R}$, and let g be a non-positive yielding partial function from X to $\mathbb{R}$. Note that $\frac{f}{g}$ is non-positive yielding.

Let X be a non empty set, let f be a positive yielding partial function from X to $\mathbb{R}$, and let g be a negative yielding partial function from X to $\mathbb{R}$. One can verify that $\frac{f}{g}$ is negative yielding.

Let X be a non empty set, let f be a negative yielding partial function from X to $\mathbb{R}$, and let g be a positive yielding partial function from X to $\mathbb{R}$. Observe that $\frac{f}{g}$ is negative yielding.

Let X be a non empty set and let f, g be positive yielding partial functions from X to $\mathbb{R}$. One can check that $\frac{f}{g}$ is positive yielding.

Let X be a non empty set and let f, g be negative yielding partial functions from X to $\mathbb{R}$. One can check that $\frac{f}{g}$ is positive yielding.

Let X be a non empty set and let f be a partial function from X to $\mathbb{R}$. Observe that $\frac{f}{f}$ is non-negative yielding.

Let X be a non empty set and let f, g be non-empty partial functions from X to $\mathbb{R}$. One can verify that $\frac{f}{g}$ is non-empty.

Let X be a set and let f be a non-positive yielding function from X into $\mathbb{R}$. One can verify that $\mathrm{Inv}\, f$ is non-positive yielding.

Let X be a set and let f be a non-negative yielding function from X into $\mathbb{R}$. Observe that $\mathrm{Inv}\, f$ is non-negative yielding.

Let X be a set and let f be a positive yielding function from X into $\mathbb{R}$. One can verify that $\mathrm{Inv}\, f$ is positive yielding.

Let X be a set and let f be a negative yielding function from X into $\mathbb{R}$. Note that $\mathrm{Inv}\, f$ is negative yielding.

Let X be a set and let f be a non-empty function from X into $\mathbb{R}$. Note that $\mathrm{Inv}\, f$ is non-empty.

Let X be a set and let f be a non-empty function from X into $\mathbb{R}$. One can verify that $-f$ is non-empty.

Let X be a set and let f be a non-positive yielding function from X into $\mathbb{R}$. Observe that $-f$ is non-negative yielding.

Let X be a set and let f be a non-negative yielding function from X into $\mathbb{R}$. One can check that $-f$ is non-positive yielding.

Let X be a set and let f be a positive yielding function from X into $\mathbb{R}$. Observe that $-f$ is negative yielding.

Let X be a set and let f be a negative yielding function from X into $\mathbb{R}$. Observe that $-f$ is positive yielding.

Let X be a set and let f be a function from X into $\mathbb{R}$. Note that $|f|$ is non-negative yielding.

Let X be a set and let f be a non-empty function from X into $\mathbb{R}$. One can check that $|f|$ is positive yielding.

Let X be a non empty set and let f be a non-positive yielding function from X into $\mathbb{R}$. Observe that $\frac{1}{f}$ is non-positive yielding.

Let X be a non empty set and let f be a non-negative yielding function from X into $\mathbb{R}$. Note that $\frac{1}{f}$ is non-negative yielding.

Let X be a non empty set and let f be a positive yielding function from X into $\mathbb{R}$. One can check that $\frac{1}{f}$ is positive yielding.

Let X be a non empty set and let f be a negative yielding function from X into $\mathbb{R}$. Note that $\frac{1}{f}$ is negative yielding.

Let X be a non empty set and let f be a non-empty function from X into $\mathbb{R}$. One can check that $\frac{1}{f}$ is non-empty.

Let f be a real-yielding function. The functor $\sqrt{f}$ yields a function and is defined as follows:

(Def. 5) $\mathrm{dom}\sqrt{f} = \mathrm{dom}\, f$ and for every set x such that $x \in \mathrm{dom}\sqrt{f}$ holds $\sqrt{f}(x) = \sqrt{f(x)}$.

Let f be a real-yielding function. Observe that $\sqrt{f}$ is real-yielding.

Let C be a set, let D be a real-membered set, and let f be a partial function from C to D. Then $\sqrt{f}$ is a partial function from C to $\mathbb{R}$.

Let X be a set and let f be a non-negative yielding function from X into $\mathbb{R}$. One can check that $\sqrt{f}$ is non-negative yielding.

Let X be a set and let f be a positive yielding function from X into $\mathbb{R}$. Note that $\sqrt{f}$ is positive yielding.

Let X be a set and let f, g be functions from X into $\mathbb{R}$. Then $f+g$ is a function from X into $\mathbb{R}$. Then $f-g$ is a function from X into $\mathbb{R}$. Then $f\,g$ is a function from X into $\mathbb{R}$.

Let X be a set and let f be a function from X into $\mathbb{R}$. Then $-f$ is a function from X into $\mathbb{R}$. Then $|f|$ is a function from X into $\mathbb{R}$. Then $\sqrt{f}$ is a function from X into $\mathbb{R}$.

Let X be a set, let f be a function from X into $\mathbb{R}$, and let r be a real number. Then $r\,f$ is a function from X into $\mathbb{R}$.

Let X be a set and let f be a non-empty function from X into $\mathbb{R}$. Then $\frac{1}{f}$ is a function from X into $\mathbb{R}$.

Let X be a non empty set, let f be a function from X into $\mathbb{R}$, and let g be a non-empty function from X into $\mathbb{R}$. Then $\frac{f}{g}$ is a function from X into $\mathbb{R}$.

In the sequel T is a non empty topological space, f, g are continuous real maps of T, and r is a real number.

Let us consider T, f, g. Then $f+g$ is a continuous real map of T. Then $f-g$ is a continuous real map of T. Then $f\,g$ is a continuous real map of T.

Let us consider T, f. Then $-f$ is a continuous real map of T.

Let us consider T, f. Then $|f|$ is a continuous real map of T.

Let us consider T. Observe that there exists a real map of T which is positive yielding and continuous and there exists a real map of T which is negative yielding and continuous.

Let us consider T and let f be a non-negative yielding continuous real map of T. Then $\sqrt{f}$ is a continuous real map of T.

Let us consider T, f, r. Then $r\,f$ is a continuous real map of T.

Let us consider T and let f be a non-empty continuous real map of T. Then $\frac{1}{f}$ is a continuous real map of T.

Let us consider T, f and let g be a non-empty continuous real map of T. Then $\frac{f}{g}$ is a continuous real map of T.

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