FORMALIZED MATHEMATICS
Volume 13, Number 3, Pages 399–406
University of Białystok, 2005

Cardinal Numbers and Finite Sets[1]

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Summary. In this paper we define class of functions and operators needed for the proof of the principle of inclusions and the disconnections. We also given certain cardinal numbers concerning elementary class of functions (this function mapping finite set in finite set).

MML identifier: `CARD_FIN`, version: `7.5.01 4.39.921`

The articles [21], [10], [24], [17], [26], [6], [27], [2], [9], [11], [1], [25], [7], [8], [22], [19], [5], [15], [12], [20], [16], [14], [18], [13], [3], [23], and [4] provide the terminology and notation for this paper.

For simplicity, we use the following convention: x, x_1, x_2, y, z, X' denote sets, X, Y denote finite sets, n, k, m denote natural numbers, and f denotes a function.

Next we state the proposition

(1) If $X \subseteq Y$ and $\operatorname{card} X = \operatorname{card} Y$, then $X = Y$.

In the sequel F is a function from $X \cup \{x\}$ into $Y \cup \{y\}$.

One can prove the following proposition

(2) For all X, Y, x, y such that if $Y = \emptyset$, then $X = \emptyset$ and $x \notin X$ holds $\operatorname{card}(Y^X) = \overline{\overline{\{F : \operatorname{rng}(F{\restriction}X) \subseteq Y \ \wedge\ F(x) = y\}}}$.

In the sequel F is a function from $X \cup \{x\}$ into Y.

One can prove the following two propositions:

(3) For all X, Y, x, y such that $x \notin X$ and $y \in Y$ holds $\operatorname{card}(Y^X) = \overline{\overline{\{F : F(x) = y\}}}$.

[1]This work has been partially supported by the KBN grant 4 T11C 039 24 and the FP6 IST grant TYPES No. 510096.

ISSN 1426–2630

(4) If if $Y = \emptyset$, then $X = \emptyset$, then $\mathrm{card}(Y^X) = (\mathrm{card}\, Y)^{\mathrm{card}\, X}$.

In the sequel F_1 denotes a function from X into Y and F_2 denotes a function from $X \cup \{x\}$ into $Y \cup \{y\}$.

One can prove the following two propositions:

(5) Let given X, Y, x, y. Suppose if Y is empty, then X is empty and $x \notin X$ and $y \notin Y$. Then $\overline{\overline{\{F_1 : F_1 \text{ is one-to-one}\}}} = \overline{\overline{\{F_2 : F_2 \text{ is one-to-one} \ \wedge \ F_2(x) = y\}}}$.

(6) $\frac{n!}{(n -' k)!}$ is a natural number.

In the sequel F is a function from X into Y.

The following proposition is true

(7) If $\mathrm{card}\, X \leq \mathrm{card}\, Y$, then $\overline{\overline{\{F : F \text{ is one-to-one}\}}} = \frac{(\mathrm{card}\, Y)!}{(\mathrm{card}\, Y -' \mathrm{card}\, X)!}$.

In the sequel F denotes a function from X into X.

The following proposition is true

(8) $\overline{\overline{\{F : F \text{ is a permutation of } X\}}} = (\mathrm{card}\, X)!$.

Let us consider X, k, x_1, x_2. The functor $\mathrm{Choose}(X, k, x_1, x_2)$ yields a subset of $\{x_1, x_2\}^X$ and is defined as follows:

(Def. 1) $x \in \mathrm{Choose}(X, k, x_1, x_2)$ iff there exists a function f from X into $\{x_1, x_2\}$ such that $f = x$ and $\overline{\overline{f^{-1}(\{x_1\})}} = k$.

We now state several propositions:

(9) If $\mathrm{card}\, X \neq k$, then $\mathrm{Choose}(X, k, x_1, x_1)$ is empty.

(10) If $\mathrm{card}\, X < k$, then $\mathrm{Choose}(X, k, x_1, x_2)$ is empty.

(11) If $x_1 \neq x_2$, then $\mathrm{card}\, \mathrm{Choose}(X, 0, x_1, x_2) = 1$.

(12) $\mathrm{card}\, \mathrm{Choose}(X, \mathrm{card}\, X, x_1, x_2) = 1$.

(13) If $f(y) = x$ and $y \in \mathrm{dom}\, f$, then $\{y\} \cup (f \restriction (\mathrm{dom}\, f \setminus \{y\}))^{-1}(\{x\}) = f^{-1}(\{x\})$.

In the sequel g denotes a function from $X \cup \{z\}$ into $\{x, y\}$.

The following propositions are true:

(14) If $z \notin X$, then $\mathrm{card}\, \mathrm{Choose}(X, k, x, y) = \overline{\overline{\{g : \overline{\overline{g^{-1}(\{x\})}} = k + 1 \ \wedge \ g(z) = x\}}}$.

(15) If $f(y) \neq x$, then $(f \restriction (\mathrm{dom}\, f \setminus \{y\}))^{-1}(\{x\}) = f^{-1}(\{x\})$.

(16) If $z \notin X$ and $x \neq y$, then $\mathrm{card}\, \mathrm{Choose}(X, k, x, y) = \overline{\overline{\{g : \overline{\overline{g^{-1}(\{x\})}} = k \ \wedge \ g(z) = y\}}}$.

(17) If $x \neq y$ and $z \notin X$, then $\mathrm{card}\, \mathrm{Choose}(X \cup \{z\}, k + 1, x, y) = \mathrm{card}\, \mathrm{Choose}(X, k + 1, x, y) + \mathrm{card}\, \mathrm{Choose}(X, k, x, y)$.

(18) If $x \neq y$, then $\mathrm{card}\, \mathrm{Choose}(X, k, x, y) = \binom{\mathrm{card}\, X}{k}$.

(19) If $x \neq y$, then $(Y \longmapsto y) {+\cdot} (X \longmapsto x) \in \mathrm{Choose}(X \cup Y, \mathrm{card}\, X, x, y)$.

(20) If $x \neq y$ and X misses Y, then $(X \longmapsto x) + \cdot (Y \longmapsto y) \in \mathrm{Choose}(X \cup Y, \mathrm{card}\, X, x, y)$.

Let F, C_1 be functions and let y be a set. The functor $\mathrm{Intersection}(F, C_1, y)$ yielding a subset of $\bigcup \mathrm{rng}\, F$ is defined as follows:

(Def. 2) $z \in \mathrm{Intersection}(F, C_1, y)$ iff $z \in \bigcup \mathrm{rng}\, F$ and for every x such that $x \in \mathrm{dom}\, C_1$ and $C_1(x) = y$ holds $z \in F(x)$.

In the sequel F, C_1 denote functions.

The following propositions are true:

(21) For all F, C_1 such that $\mathrm{dom}\, F \cap {C_1}^{-1}(\{x\})$ is non empty holds $y \in \mathrm{Intersection}(F, C_1, x)$ iff for every z such that $z \in \mathrm{dom}\, C_1$ and $C_1(z) = x$ holds $y \in F(z)$.

(22) If $\mathrm{Intersection}(F, C_1, y)$ is non empty, then ${C_1}^{-1}(\{y\}) \subseteq \mathrm{dom}\, F$.

(23) If $\mathrm{Intersection}(F, C_1, y)$ is non empty, then for all x_1, x_2 such that $x_1 \in {C_1}^{-1}(\{y\})$ and $x_2 \in {C_1}^{-1}(\{y\})$ holds $F(x_1)$ meets $F(x_2)$.

(24) If $z \in \mathrm{Intersection}(F, C_1, y)$ and $y \in \mathrm{rng}\, C_1$, then there exists x such that $x \in \mathrm{dom}\, C_1$ and $C_1(x) = y$ and $z \in F(x)$.

(25) If F is empty or $\bigcup \mathrm{rng}\, F$ is empty, then $\mathrm{Intersection}(F, C_1, y) = \bigcup \mathrm{rng}\, F$.

(26) If $F \restriction {C_1}^{-1}(\{y\}) = {C_1}^{-1}(\{y\}) \longmapsto \bigcup \mathrm{rng}\, F$, then $\mathrm{Intersection}(F, C_1, y) = \bigcup \mathrm{rng}\, F$.

(27) If $\bigcup \mathrm{rng}\, F$ is non empty and $\mathrm{Intersection}(F, C_1, y) = \bigcup \mathrm{rng}\, F$, then $F \restriction {C_1}^{-1}(\{y\}) = {C_1}^{-1}(\{y\}) \longmapsto \bigcup \mathrm{rng}\, F$.

(28) $\mathrm{Intersection}(F, \emptyset, y) = \bigcup \mathrm{rng}\, F$.

(29) $\mathrm{Intersection}(F, C_1, y) \subseteq \mathrm{Intersection}(F, C_1 \restriction X', y)$.

(30) If ${C_1}^{-1}(\{y\}) = (C_1 \restriction X')^{-1}(\{y\})$, then $\mathrm{Intersection}(F, C_1, y) = \mathrm{Intersection}(F, C_1 \restriction X', y)$.

(31) $\mathrm{Intersection}(F \restriction X', C_1, y) \subseteq \mathrm{Intersection}(F, C_1, y)$.

(32) If $y \in \mathrm{rng}\, C_1$ and ${C_1}^{-1}(\{y\}) \subseteq X'$, then $\mathrm{Intersection}(F \restriction X', C_1, y) = \mathrm{Intersection}(F, C_1, y)$.

(33) If $x \in {C_1}^{-1}(\{y\})$, then $\mathrm{Intersection}(F, C_1, y) \subseteq F(x)$.

(34) If $x \in {C_1}^{-1}(\{y\})$, then $\mathrm{Intersection}(F, C_1 \restriction (\mathrm{dom}\, C_1 \setminus \{x\}), y) \cap F(x) = \mathrm{Intersection}(F, C_1, y)$.

(35) For all functions C_2, C_3 such that ${C_2}^{-1}(\{x_1\}) = {C_3}^{-1}(\{x_2\})$ holds $\mathrm{Intersection}(F, C_2, x_1) = \mathrm{Intersection}(F, C_3, x_2)$.

(36) If ${C_1}^{-1}(\{y\}) = \emptyset$, then $\mathrm{Intersection}(F, C_1, y) = \bigcup \mathrm{rng}\, F$.

(37) If $\{x\} = {C_1}^{-1}(\{y\})$, then $\mathrm{Intersection}(F, C_1, y) = F(x)$.

(38) If $\{x_1, x_2\} = {C_1}^{-1}(\{y\})$, then $\mathrm{Intersection}(F, C_1, y) = F(x_1) \cap F(x_2)$.

(39) For every F such that F is non empty holds $y \in \mathrm{Intersection}(F, \mathrm{dom}\, F \longmapsto x, x)$ iff for every z such that $z \in \mathrm{dom}\, F$ holds $y \in F(z)$.

Let F be a function. We say that F is finite-yielding if and only if:

(Def. 3) For every x holds $F(x)$ is finite.

Let us observe that there exists a function which is non empty and finite-yielding and there exists a function which is empty and finite-yielding.

Let F be a finite-yielding function and let x be a set. Observe that $F(x)$ is finite.

Let F be a finite-yielding function and let X be a set. One can check that $F{\restriction}X$ is finite-yielding.

Let F be a finite-yielding function and let G be a function. Note that $F \cdot G$ is finite-yielding and $\mathrm{Intersect}(F, G)$ is finite-yielding.

In the sequel F_3 is a finite-yielding function.

The following two propositions are true:

(40) If $y \in \mathrm{rng}\, C_1$, then $\mathrm{Intersection}(F_3, C_1, y)$ is finite.

(41) If $\mathrm{dom}\, F_3$ is finite, then $\bigcup \mathrm{rng}\, F_3$ is finite.

Let F be a finite 0-sequence and let us consider n. Then $F{\restriction}n$ is a finite 0-sequence.

Let D be a set, let F be a finite 0-sequence of D, and let us consider n. Then $F{\restriction}n$ is a finite 0-sequence of D.

In the sequel D is a non empty set and b is a binary operation on D.

Next we state several propositions:

(42) For every finite 0-sequence F of D and for all b, n such that $n \in \mathrm{dom}\, F$ but b has a unity or $n \neq 0$ holds $b(b \odot F{\restriction}n,\ F(n)) = b \odot F{\restriction}(n + 1)$.

(43) For every finite 0-sequence F of D and for every n such that $\mathrm{len}\, F = n+1$ holds $F = (F{\restriction}n) \frown \langle F(n) \rangle$.

(44) For every finite 0-sequence F of $\mathbb{N}$ and for every n such that $n \in \mathrm{dom}\, F$ holds $\sum(F{\restriction}n) + F(n) = \sum(F{\restriction}(n + 1))$.

(45) For every finite 0-sequence F of $\mathbb{N}$ and for every n such that $\mathrm{rng}\, F \subseteq \{0, n\}$ holds $\sum F = n \cdot \mathrm{card}(F^{-1}(\{n\}))$.

(46) $x \in \mathrm{Choose}(n, k, 1, 0)$ iff there exists a finite 0-sequence F of $\mathbb{N}$ such that $F = x$ and $\mathrm{dom}\, F = n$ and $\mathrm{rng}\, F \subseteq \{0, 1\}$ and $\sum F = k$.

(47) For every finite 0-sequence F of D and for every b such that b has a unity or $\mathrm{len}\, F \geq 1$ holds $b \odot F = b \odot \mathrm{XFS2FS}(F)$.

(48) Let F, G be finite 0-sequences of D and P be a permutation of $\mathrm{dom}\, F$. Suppose b is commutative and associative but b has a unity or $\mathrm{len}\, F \geq 1$ but $G = F \cdot P$. Then $b \odot F = b \odot G$.

Let us consider k and let F be a finite-yielding function. Let us assume that $\mathrm{dom}\, F$ is finite. The card intersection of F wrt k yielding a natural number is defined by the condition (Def. 4).

(Def. 4) Let x, y be sets, X be a finite set, and P be a function from $\mathrm{card}\, \mathrm{Choose}(X, k, x, y)$ into $\mathrm{Choose}(X, k, x, y)$. Suppose $\mathrm{dom}\, F = X$ and

P is one-to-one and $x \neq y$. Then there exists a finite 0-sequence X_1 of $\mathbb{N}$ such that $\operatorname{dom} X_1 = \operatorname{dom} P$ and for all z, f such that $z \in \operatorname{dom} X_1$ and $f = P(z)$ holds $X_1(z) = \overline{\overline{\operatorname{Intersection}(F, f, x)}}$ and the card intersection of F wrt $k = \sum X_1$.

One can prove the following propositions:

(49) Let x, y be sets, X be a finite set, and P be a function from $\operatorname{card} \operatorname{Choose}(X, k, x, y)$ into $\operatorname{Choose}(X, k, x, y)$. Suppose $\operatorname{dom} F_3 = X$ and P is one-to-one and $x \neq y$. Let X_1 be a finite 0-sequence of $\mathbb{N}$. Suppose $\operatorname{dom} X_1 = \operatorname{dom} P$ and for all z, f such that $z \in \operatorname{dom} X_1$ and $f = P(z)$ holds $X_1(z) = \overline{\overline{\operatorname{Intersection}(F_3, f, x)}}$. Then the card intersection of F_3 wrt $k = \sum X_1$.

(50) If $\operatorname{dom} F_3$ is finite and $k = 0$, then the card intersection of F_3 wrt $k = \overline{\overline{\bigcup \operatorname{rng} F_3}}$.

(51) If $\operatorname{dom} F_3 = X$ and $k > \operatorname{card} X$, then the card intersection of F_3 wrt $k = 0$.

(52) Let given F_3, X. Suppose $\operatorname{dom} F_3 = X$. Let P be a function from $\operatorname{card} X$ into X. Suppose P is one-to-one. Then there exists a finite 0-sequence X_1 of $\mathbb{N}$ such that $\operatorname{dom} X_1 = \operatorname{card} X$ and for every z such that $z \in \operatorname{dom} X_1$ holds $X_1(z) = \operatorname{card}(F_3 \cdot P)(z)$ and the card intersection of F_3 wrt $1 = \sum X_1$.

(53) If $\operatorname{dom} F_3 = X$, then the card intersection of F_3 wrt $\operatorname{card} X = \overline{\overline{\operatorname{Intersection}(F_3, X \longmapsto x, x)}}$.

(54) If $F_3 = \{x\} \longmapsto X$, then the card intersection of F_3 wrt $1 = \operatorname{card} X$.

(55) Suppose $x \neq y$ and $F_3 = [x \longmapsto X, y \longmapsto Y]$. Then the card intersection of F_3 wrt $1 = \operatorname{card} X + \operatorname{card} Y$ and the card intersection of F_3 wrt $2 = \operatorname{card}(X \cap Y)$.

(56) Let given F_3, x. Suppose $\operatorname{dom} F_3$ is finite and $x \in \operatorname{dom} F_3$. Then the card intersection of F_3 wrt $1 =$ (the card intersection of $F_3 \restriction (\operatorname{dom} F_3 \setminus \{x\})$ wrt 1)$+ \operatorname{card} F_3(x)$.

(57) $\operatorname{dom} \operatorname{Intersect}(F, \operatorname{dom} F \longmapsto X') = \operatorname{dom} F$ and for every x such that $x \in \operatorname{dom} F$ holds $(\operatorname{Intersect}(F, \operatorname{dom} F \longmapsto X'))(x) = F(x) \cap X'$.

(58) $\bigcup \operatorname{rng} F \cap X' = \bigcup \operatorname{rng} \operatorname{Intersect}(F, \operatorname{dom} F \longmapsto X')$.

(59) $\operatorname{Intersection}(F, C_1, y) \cap X' = \operatorname{Intersection}(\operatorname{Intersect}(F, \operatorname{dom} F \longmapsto X'), C_1, y)$.

(60) Let F, G be finite 0-sequences. Suppose F is one-to-one and G is one-to-one and $\operatorname{rng} F$ misses $\operatorname{rng} G$. Then $F \frown G$ is one-to-one.

(61) Let given F_3, X, x, n. Suppose $\operatorname{dom} F_3 = X$ and $x \in \operatorname{dom} F_3$ and $k > 0$. Then the card intersection of F_3 wrt $k + 1 =$ (the card intersection of $F_3 \restriction (\operatorname{dom} F_3 \setminus \{x\})$ wrt $k + 1$) $+$ (the card intersection of

$\mathrm{Intersect}(F_3 \restriction (\mathrm{dom}\, F_3 \setminus \{x\}), \mathrm{dom}\, F_3 \setminus \{x\} \longmapsto F_3(x))$ wrt k).

(62) Let F, G, b_1 be finite 0-sequences of D. Suppose that

(i) b is commutative and associative,

(ii) b has a unity or $\mathrm{len}\, F \geq 1$,

(iii) $\mathrm{len}\, F = \mathrm{len}\, G$,

(iv) $\mathrm{len}\, F = \mathrm{len}\, b_1$, and

(v) for every n such that $n \in \mathrm{dom}\, b_1$ holds $b_1(n) = b(F(n), G(n))$.

Then $b \odot F \frown G = b \odot b_1$.

Let F_4 be a finite 0-sequence of $\mathbb{Z}$. The functor $\sum F_4$ yielding an integer is defined as follows:

(Def. 5) $\sum F_4 = +_{\mathbb{Z}} \odot F_4$.

Let F_4 be a finite 0-sequence of $\mathbb{Z}$ and let us consider x. Then $F_4(x)$ is an integer.

Next we state several propositions:

(63) For every finite 0-sequence F_5 of $\mathbb{N}$ and for every finite 0-sequence F_4 of $\mathbb{Z}$ such that $F_4 = F_5$ holds $\sum F_4 = \sum F_5$.

(64) Let F, F_4 be finite 0-sequences of $\mathbb{Z}$ and i be an integer. If $\mathrm{dom}\, F = \mathrm{dom}\, F_4$ and for every n such that $n \in \mathrm{dom}\, F$ holds $i \cdot F(n) = F_4(n)$, then $i \cdot \sum F = \sum F_4$.

(65) If $x \in \mathrm{dom}\, F$, then $\bigcup \mathrm{rng}\, F = \bigcup \mathrm{rng}(F \restriction (\mathrm{dom}\, F \setminus \{x\})) \cup F(x)$.

(66) Let F_3 be a finite-yielding function and given X. Then there exists a finite 0-sequence X_1 of $\mathbb{Z}$ such that $\mathrm{dom}\, X_1 = \mathrm{card}\, X$ and for every n such that $n \in \mathrm{dom}\, X_1$ holds $X_1(n) = (-1)^n \cdot$ the card intersection of F_3 wrt $n+1$.

(67) Let F_3 be a finite-yielding function and given X. Suppose $\mathrm{dom}\, F_3 = X$. Let X_1 be a finite 0-sequence of $\mathbb{Z}$. Suppose $\mathrm{dom}\, X_1 = \mathrm{card}\, X$ and for every n such that $n \in \mathrm{dom}\, X_1$ holds $X_1(n) = (-1)^n \cdot$ the card intersection of F_3 wrt $n+1$. Then $\overline{\overline{\bigcup \mathrm{rng}\, F_3}} = \sum X_1$.

(68) Let given F_3, X, n, k. Suppose $\mathrm{dom}\, F_3 = X$. Given x, y such that $x \neq y$ and for every f such that $f \in \mathrm{Choose}(X, k, x, y)$ holds $\overline{\overline{\mathrm{Intersection}(F_3, f, x)}} = n$. Then the card intersection of F_3 wrt $k = n \cdot \binom{\mathrm{card}\, X}{k}$.

(69) Let given F_3, X. Suppose $\mathrm{dom}\, F_3 = X$. Let X_2 be a finite 0-sequence of $\mathbb{N}$. Suppose $\mathrm{dom}\, X_2 = \mathrm{card}\, X$ and for every n such that $n \in \mathrm{dom}\, X_2$ there exist x, y such that $x \neq y$ and for every f such that $f \in \mathrm{Choose}(X, n+1, x, y)$ holds $\overline{\overline{\mathrm{Intersection}(F_3, f, x)}} = X_2(n)$. Then there exists a finite 0-sequence F of $\mathbb{Z}$ such that $\mathrm{dom}\, F = \mathrm{card}\, X$ and $\overline{\overline{\bigcup \mathrm{rng}\, F_3}} = \sum F$ and for every n such that $n \in \mathrm{dom}\, F$ holds $F(n) = (-1)^n \cdot X_2(n) \cdot \binom{\mathrm{card}\, X}{n+1}$.

In the sequel g denotes a function from X into Y.

The following propositions are true:

(70) Let X, Y be finite sets. Suppose X is non empty and Y is non empty. Then there exists a finite 0-sequence F of $\mathbb{Z}$ such that $\operatorname{dom} F = \operatorname{card} Y + 1$ and $\sum F = \overline{\overline{\{g : g \text{ is onto}\}}}$ and for every n such that $n \in \operatorname{dom} F$ holds $F(n) = (-1)^n \cdot \binom{\operatorname{card} Y}{n} \cdot (\operatorname{card} Y - n)^{\operatorname{card} X}$.

(71) Let given n, k. Suppose $k \leq n$. Then there exists a finite 0-sequence F of $\mathbb{Z}$ such that $n \operatorname{block} k = \frac{1}{k!} \cdot \sum F$ and $\operatorname{dom} F = k + 1$ and for every m such that $m \in \operatorname{dom} F$ holds $F(m) = (-1)^m \cdot \binom{k}{m} \cdot (k - m)^n$.

In the sequel A, B are finite sets and f is a function from A into B.

One can prove the following proposition

(72) Let given A, B and X be a finite set. Suppose if B is empty, then A is empty and $X \subseteq A$. Let F be a function from A into B. Suppose F is one-to-one and $\operatorname{card} A = \operatorname{card} B$. Then $(\operatorname{card} A -' \operatorname{card} X)! = \overline{\overline{\{f : f \text{ is one-to-one } \wedge \operatorname{rng}(f{\restriction}(A \setminus X)) \subseteq F^{\circ}(A \setminus X) \wedge \bigwedge_x (x \in X \Rightarrow f(x) = F(x))\}}}$.

In the sequel F denotes a function and h denotes a function from X into $\operatorname{rng} F$.

The following proposition is true

(73) Let given F. Suppose $\operatorname{dom} F = X$ and F is one-to-one. Then there exists a finite 0-sequence X_2 of $\mathbb{Z}$ such that

(i) $\sum X_2 = \overline{\overline{\{h : h \text{ is one-to-one } \wedge \bigwedge_x (x \in X \Rightarrow h(x) \neq F(x))\}}}$,

(ii) $\operatorname{dom} X_2 = \operatorname{card} X + 1$, and

(iii) for every n such that $n \in \operatorname{dom} X_2$ holds $X_2(n) = \frac{(-1)^n \cdot (\operatorname{card} X)!}{n!}$.

In the sequel h is a function from X into X.

The following proposition is true

(74) There exists a finite 0-sequence X_2 of $\mathbb{Z}$ such that

(i) $\sum X_2 = \overline{\overline{\{h : h \text{ is one-to-one } \wedge \bigwedge_x (x \in X \Rightarrow h(x) \neq x)\}}}$,

(ii) $\operatorname{dom} X_2 = \operatorname{card} X + 1$, and

(iii) for every n such that $n \in \operatorname{dom} X_2$ holds $X_2(n) = \frac{(-1)^n \cdot (\operatorname{card} X)!}{n!}$.

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Received May 24, 2005