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Some Equations Related to the Limit of Sequence of Subsets

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Summary. Set operations for sequences of subsets are introduced here. Some relations for these operations with the limit of sequences of subsets, also with the inferior sequence and the superior sequence of sets, and with the inferior limit and the superior limit of sets are shown.

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The articles [5], [2], [6], [1], [3], [4], and [7] provide the notation and terminology for this paper.

For simplicity, we use the following convention: n, k denote natural numbers, X denotes a set, A denotes a subset of X, and A_1, A_2 denote sequences of subsets of X.

We now state two propositions:

(1) (The inferior setsequence $A_1)(n) = \mathrm{Intersection}(A_1 \uparrow n)$.

(2) (The superior setsequence $A_1)(n) = \bigcup(A_1 \uparrow n)$.

Let us consider X and let A_1, A_2 be sequences of subsets of X. The functor $A_1 \cap A_2$ yields a sequence of subsets of X and is defined as follows:

(Def. 1) For every n holds $(A_1 \cap A_2)(n) = A_1(n) \cap A_2(n)$.

Let us note that the functor $A_1 \cap A_2$ is commutative. The functor $A_1 \cup A_2$ yielding a sequence of subsets of X is defined as follows:

(Def. 2) For every n holds $(A_1 \cup A_2)(n) = A_1(n) \cup A_2(n)$.

Let us observe that the functor $A_1 \cup A_2$ is commutative. The functor $A_1 \setminus A_2$ yielding a sequence of subsets of X is defined by:

(Def. 3) For every n holds $(A_1 \setminus A_2)(n) = A_1(n) \setminus A_2(n)$.

The functor $A_1 \dot{-} A_2$ yields a sequence of subsets of X and is defined as follows:

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(Def. 4) For every n holds $(A_1 \dot{-} A_2)(n) = A_1(n) \dot{-} A_2(n)$.

Let us note that the functor $A_1 \dot{-} A_2$ is commutative.

One can prove the following propositions:

(3) $A_1 \dot{-} A_2 = (A_1 \setminus A_2) \cup (A_2 \setminus A_1)$.

(4) $(A_1 \cap A_2) \uparrow k = A_1 \uparrow k \cap A_2 \uparrow k$.

(5) $(A_1 \cup A_2) \uparrow k = A_1 \uparrow k \cup A_2 \uparrow k$.

(6) $(A_1 \setminus A_2) \uparrow k = A_1 \uparrow k \setminus A_2 \uparrow k$.

(7) $(A_1 \dot{-} A_2) \uparrow k = A_1 \uparrow k \dot{-} A_2 \uparrow k$.

(8) $\bigcup(A_1 \cap A_2) \subseteq \bigcup A_1 \cap \bigcup A_2$.

(9) $\bigcup(A_1 \cup A_2) = \bigcup A_1 \cup \bigcup A_2$.

(10) $\bigcup A_1 \setminus \bigcup A_2 \subseteq \bigcup(A_1 \setminus A_2)$.

(11) $\bigcup A_1 \dot{-} \bigcup A_2 \subseteq \bigcup(A_1 \dot{-} A_2)$.

(12) $\mathrm{Intersection}(A_1 \cap A_2) = \mathrm{Intersection}\, A_1 \cap \mathrm{Intersection}\, A_2$.

(13) $\mathrm{Intersection}\, A_1 \cup \mathrm{Intersection}\, A_2 \subseteq \mathrm{Intersection}(A_1 \cup A_2)$.

(14) $\mathrm{Intersection}(A_1 \setminus A_2) \subseteq \mathrm{Intersection}\, A_1 \setminus \mathrm{Intersection}\, A_2$.

Let us consider X, let A_1 be a sequence of subsets of X, and let A be a subset of X. The functor $A \cap A_1$ yielding a sequence of subsets of X is defined by:

(Def. 5) For every n holds $(A \cap A_1)(n) = A \cap A_1(n)$.

The functor $A \cup A_1$ yielding a sequence of subsets of X is defined as follows:

(Def. 6) For every n holds $(A \cup A_1)(n) = A \cup A_1(n)$.

The functor $A \setminus A_1$ yields a sequence of subsets of X and is defined by:

(Def. 7) For every n holds $(A \setminus A_1)(n) = A \setminus A_1(n)$.

The functor $A_1 \setminus A$ yields a sequence of subsets of X and is defined by:

(Def. 8) For every n holds $(A_1 \setminus A)(n) = A_1(n) \setminus A$.

The functor $A \dot{-} A_1$ yielding a sequence of subsets of X is defined as follows:

(Def. 9) For every n holds $(A \dot{-} A_1)(n) = A \dot{-} A_1(n)$.

One can prove the following propositions:

(15) $A \dot{-} A_1 = (A \setminus A_1) \cup (A_1 \setminus A)$.

(16) $(A \cap A_1) \uparrow k = A \cap A_1 \uparrow k$.

(17) $(A \cup A_1) \uparrow k = A \cup A_1 \uparrow k$.

(18) $(A \setminus A_1) \uparrow k = A \setminus A_1 \uparrow k$.

(19) $(A_1 \setminus A) \uparrow k = A_1 \uparrow k \setminus A$.

(20) $(A \dot{-} A_1) \uparrow k = A \dot{-} A_1 \uparrow k$.

(21) If A_1 is non-increasing, then $A \cap A_1$ is non-increasing.

(22) If A_1 is non-decreasing, then $A \cap A_1$ is non-decreasing.

(23) If A_1 is monotone, then $A \cap A_1$ is monotone.

(24) If A_1 is non-increasing, then $A \cup A_1$ is non-increasing.

(25) If A_1 is non-decreasing, then $A \cup A_1$ is non-decreasing.

(26) If A_1 is monotone, then $A \cup A_1$ is monotone.

(27) If A_1 is non-increasing, then $A \setminus A_1$ is non-decreasing.

(28) If A_1 is non-decreasing, then $A \setminus A_1$ is non-increasing.

(29) If A_1 is monotone, then $A \setminus A_1$ is monotone.

(30) If A_1 is non-increasing, then $A_1 \setminus A$ is non-increasing.

(31) If A_1 is non-decreasing, then $A_1 \setminus A$ is non-decreasing.

(32) If A_1 is monotone, then $A_1 \setminus A$ is monotone.

(33) $\mathrm{Intersection}(A \cap A_1) = A \cap \mathrm{Intersection}\, A_1$.

(34) $\mathrm{Intersection}(A \cup A_1) = A \cup \mathrm{Intersection}\, A_1$.

(35) $\mathrm{Intersection}(A \setminus A_1) \subseteq A \setminus \mathrm{Intersection}\, A_1$.

(36) $\mathrm{Intersection}(A_1 \setminus A) = \mathrm{Intersection}\, A_1 \setminus A$.

(37) $\mathrm{Intersection}(A \dot{-} A_1) \subseteq A \dot{-} \mathrm{Intersection}\, A_1$.

(38) $\bigcup(A \cap A_1) = A \cap \bigcup A_1$.

(39) $\bigcup(A \cup A_1) = A \cup \bigcup A_1$.

(40) $A \setminus \bigcup A_1 \subseteq \bigcup(A \setminus A_1)$.

(41) $\bigcup(A_1 \setminus A) = \bigcup A_1 \setminus A$.

(42) $A \dot{-} \bigcup A_1 \subseteq \bigcup(A \dot{-} A_1)$.

(43) (The inferior setsequence $A_1 \cap A_2)(n)$ = (the inferior setsequence $A_1)(n) \cap$ (the inferior setsequence $A_2)(n)$.

(44) (The inferior setsequence $A_1)(n) \cup$(the inferior setsequence $A_2)(n) \subseteq$ (the inferior setsequence $A_1 \cup A_2)(n)$.

(45) (The inferior setsequence $A_1 \setminus A_2)(n) \subseteq$ (the inferior setsequence $A_1)(n) \setminus$ (the inferior setsequence $A_2)(n)$.

(46) (The superior setsequence $A_1 \cap A_2)(n) \subseteq$ (the superior setsequence $A_1)(n) \cap$ (the superior setsequence $A_2)(n)$.

(47) (The superior setsequence $A_1 \cup A_2)(n)$ = (the superior setsequence $A_1)(n) \cup$ (the superior setsequence $A_2)(n)$.

(48) (The superior setsequence $A_1)(n) \setminus$ (the superior setsequence $A_2)(n) \subseteq$ (the superior setsequence $A_1 \setminus A_2)(n)$.

(49) (The superior setsequence $A_1)(n) \dot{-}$(the superior setsequence $A_2)(n) \subseteq$ (the superior setsequence $A_1 \dot{-} A_2)(n)$.

(50) (The inferior setsequence $A \cap A_1)(n) = A \cap$ (the inferior setsequence $A_1)(n)$.

(51) (The inferior setsequence $A \cup A_1)(n) = A \cup$ (the inferior setsequence $A_1)(n)$.

(52) (The inferior setsequence $A \setminus A_1)(n) \subseteq A \setminus$ (the inferior setsequence $A_1)(n)$.

(53) (The inferior setsequence $A_1 \setminus A)(n) =$ (the inferior setsequence $A_1)(n) \setminus A$.

(54) (The inferior setsequence $A \dot{-} A_1)(n) \subseteq A \dot{-}$(the inferior setsequence $A_1)(n)$.

(55) (The superior setsequence $A \cap A_1)(n) = A \cap$ (the superior setsequence $A_1)(n)$.

(56) (The superior setsequence $A \cup A_1)(n) = A \cup$ (the superior setsequence $A_1)(n)$.

(57) $A \setminus$ (the superior setsequence $A_1)(n) \subseteq$ (the superior setsequence $A \setminus A_1)(n)$.

(58) (The superior setsequence $A_1 \setminus A)(n) =$ (the superior setsequence $A_1)(n) \setminus A$.

(59) $A \dot{-}$(the superior setsequence $A_1)(n) \subseteq$ (the superior setsequence $A \dot{-} A_1)(n)$.

(60) $\liminf(A_1 \cap A_2) = \liminf A_1 \cap \liminf A_2$.

(61) $\liminf A_1 \cup \liminf A_2 \subseteq \liminf(A_1 \cup A_2)$.

(62) $\liminf(A_1 \setminus A_2) \subseteq \liminf A_1 \setminus \liminf A_2$.

(63) If A_1 is convergent or A_2 is convergent, then $\liminf(A_1 \cup A_2) = \liminf A_1 \cup \liminf A_2$.

(64) If A_2 is convergent, then $\liminf(A_1 \setminus A_2) = \liminf A_1 \setminus \liminf A_2$.

(65) If A_1 is convergent or A_2 is convergent, then $\liminf(A_1 \dot{-} A_2) \subseteq \liminf A_1 \dot{-} \liminf A_2$.

(66) If A_1 is convergent and A_2 is convergent, then $\liminf(A_1 \dot{-} A_2) = \liminf A_1 \dot{-} \liminf A_2$.

(67) $\limsup(A_1 \cap A_2) \subseteq \limsup A_1 \cap \limsup A_2$.

(68) $\limsup(A_1 \cup A_2) = \limsup A_1 \cup \limsup A_2$.

(69) $\limsup A_1 \setminus \limsup A_2 \subseteq \limsup(A_1 \setminus A_2)$.

(70) $\limsup A_1 \dot{-} \limsup A_2 \subseteq \limsup(A_1 \dot{-} A_2)$.

(71) If A_1 is convergent or A_2 is convergent, then $\limsup(A_1 \cap A_2) = \limsup A_1 \cap \limsup A_2$.

(72) If A_2 is convergent, then $\limsup(A_1 \setminus A_2) = \limsup A_1 \setminus \limsup A_2$.

(73) If A_1 is convergent and A_2 is convergent, then $\limsup(A_1 \dot{-} A_2) = \limsup A_1 \dot{-} \limsup A_2$.

(74) $\liminf(A \cap A_1) = A \cap \liminf A_1$.

(75) $\liminf(A \cup A_1) = A \cup \liminf A_1$.

(76) $\liminf(A \setminus A_1) \subseteq A \setminus \liminf A_1$.

(77) $\liminf(A_1 \setminus A) = \liminf A_1 \setminus A$.

(78) $\liminf(A \dot{-} A_1) \subseteq A \dot{-} \liminf A_1$.

(79) If A_1 is convergent, then $\liminf(A \setminus A_1) = A \setminus \liminf A_1$.

(80) If A_1 is convergent, then $\liminf(A \dot{-} A_1) = A \dot{-} \liminf A_1$.

(81) $\limsup(A \cap A_1) = A \cap \limsup A_1$.

(82) $\limsup(A \cup A_1) = A \cup \limsup A_1$.

(83) $A \setminus \limsup A_1 \subseteq \limsup(A \setminus A_1)$.

(84) $\limsup(A_1 \setminus A) = \limsup A_1 \setminus A$.

(85) $A \dot{-} \limsup A_1 \subseteq \limsup(A \dot{-} A_1)$.

(86) If A_1 is convergent, then $\limsup(A \setminus A_1) = A \setminus \limsup A_1$.

(87) If A_1 is convergent, then $\limsup(A \dot{-} A_1) = A \dot{-} \limsup A_1$.

(88) If A_1 is convergent and A_2 is convergent, then $A_1 \cap A_2$ is convergent and $\lim(A_1 \cap A_2) = \lim A_1 \cap \lim A_2$.

(89) If A_1 is convergent and A_2 is convergent, then $A_1 \cup A_2$ is convergent and $\lim(A_1 \cup A_2) = \lim A_1 \cup \lim A_2$.

(90) If A_1 is convergent and A_2 is convergent, then $A_1 \setminus A_2$ is convergent and $\lim(A_1 \setminus A_2) = \lim A_1 \setminus \lim A_2$.

(91) If A_1 is convergent and A_2 is convergent, then $A_1 \dot{-} A_2$ is convergent and $\lim(A_1 \dot{-} A_2) = \lim A_1 \dot{-} \lim A_2$.

(92) If A_1 is convergent, then $A \cap A_1$ is convergent and $\lim(A \cap A_1) = A \cap \lim A_1$.

(93) If A_1 is convergent, then $A \cup A_1$ is convergent and $\lim(A \cup A_1) = A \cup \lim A_1$.

(94) If A_1 is convergent, then $A \setminus A_1$ is convergent and $\lim(A \setminus A_1) = A \setminus \lim A_1$.

(95) If A_1 is convergent, then $A_1 \setminus A$ is convergent and $\lim(A_1 \setminus A) = \lim A_1 \setminus A$.

(96) If A_1 is convergent, then $A \dot{-} A_1$ is convergent and $\lim(A \dot{-} A_1) = A \dot{-} \lim A_1$.

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