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Homeomorphisms of Jordan Curves

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Summary. In this paper we prove that simple closed curves can be homeomorphically framed into a given rectangle. We also show that homeomorphisms preserve the Jordan property.

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The notation and terminology used in this paper are introduced in the following articles: [20], [21], [1], [3], [22], [4], [5], [19], [10], [18], [7], [17], [11], [2], [8], [9], [16], [13], [14], [15], [6], [23], and [12].

In this paper p_1, p_2 are points of $\mathcal{E}^2_{\rm T}$, C is a simple closed curve, and P is a subset of $\mathcal{E}^2_{\rm T}$.

Let n be a natural number, let A be a subset of $\mathcal{E}^n_{\rm T}$, and let a, b be points of $\mathcal{E}^n_{\rm T}$. We say that a and b realize maximal distance in A if and only if:

(Def. 1) $a \in A$ and $b \in A$ and for all points x, y of $\mathcal{E}^n_{\rm T}$ such that $x \in A$ and $y \in A$ holds $\rho(a,b) \geq \rho(x,y)$.

Next we state the proposition

(1) There exist p_1, p_2 such that p_1 and p_2 realize maximal distance in C.

Let M be a non empty metric structure and let f be a map from $M_{\rm top}$ into $M_{\rm top}$. We say that f is isometric if and only if:

(Def. 2) There exists an isometric map g from M into M such that $g = f$.

Let M be a non empty metric structure. Note that there exists a map from $M_{\rm top}$ into $M_{\rm top}$ which is isometric.

Let M be a non empty metric space. Observe that every map from $M_{\rm top}$ into $M_{\rm top}$ which is isometric is also continuous.

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Let M be a non empty metric space. Note that every map from $M_{\rm top}$ into $M_{\rm top}$ which is isometric is also homeomorphism.

Let a be a real number. The functor Rotate a yields a map from $\mathcal{E}^2_{\rm T}$ into $\mathcal{E}^2_{\rm T}$ and is defined as follows:

(Def. 3) For every point p of $\mathcal{E}^2_{\rm T}$ holds $({\rm Rotate}\,a)(p) = [\Re(\mathbf{p_1} + \mathbf{p_2} \cdot i \circlearrowleft a), \Im(\mathbf{p_1} + \mathbf{p_2} \cdot i \circlearrowleft a)]$, where $a = [r_1, 0]$ and $r_1 = -1$.

The following propositions are true:

(2) Let a be a real number. Suppose $0 \leq a$ and $a < 2 \cdot \pi$. Let f be a map from $(\mathcal{E}^2)_{\rm top}$ into $(\mathcal{E}^2)_{\rm top}$. If $f = {\rm Rotate}\,a$, then f is isometric, where $a = [r_1, 0]$ and $r_1 = -1$.

(3) Let A, B, D be real numbers. Suppose p_1 and p_2 realize maximal distance in P. Then $({\rm AffineMap}(A, B, A, D))(p_1)$ and $({\rm AffineMap}(A, B, A, D))(p_2)$ realize maximal distance in $({\rm AffineMap}(A, B, A, D))^\circ P$.

(4) Let A be a real number. Suppose $0 \leq A$ and $A < 2 \cdot \pi$ and p_1 and p_2 realize maximal distance in P. Then $({\rm Rotate}\,A)(p_1)$ and $({\rm Rotate}\,A)(p_2)$ realize maximal distance in $({\rm Rotate}\,A)^\circ P$.

(5) For every complex number z and for every real number r holds $z \circlearrowleft -r = z \circlearrowleft 2 \cdot \pi - r$.

(6) For every real number r holds ${\rm Rotate}(-r) = {\rm Rotate}(2 \cdot \pi - r)$.

(7) There exists a homeomorphism f of $\mathcal{E}^2_{\rm T}$ such that $[-1, 0]$ and $[1, 0]$ realize maximal distance in $f^\circ C$.

Let T_1, T_2 be topological structures and let f be a map from T_1 into T_2. We say that f is closed if and only if:

(Def. 4) For every subset A of T_1 such that A is closed holds $f^\circ A$ is closed.

One can prove the following propositions:

(8) Let X, Y be non empty topological spaces and f be a continuous map from X into Y. Suppose f is one-to-one and onto. Then f is a homeomorphism if and only if f is closed.

(9) For every set X and for every subset A of X holds $A^{\rm c} = \emptyset$ iff $A = X$.

(10) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1. If A is connected, then $f^\circ A$ is connected.

(11) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1. If A is a component of T_1, then $f^\circ A$ is a component of T_2.

(12) Let T_1, T_2 be non empty topological spaces, f be a map from T_1 into T_2, and A be a subset of T_1. Then $f \restriction A$ is a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$.

(13) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into

T_2. Suppose f is continuous. Let A be a subset of T_1 and g be a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$. If $g = f \restriction A$, then g is continuous.

(14) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1 and g be a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$. If $g = f \restriction A$, then g is a homeomorphism.

(15) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A, B be subsets of T_1. If A is a component of B, then $f^\circ A$ is a component of $f^\circ B$.

(16) For every subset S of $\mathcal{E}^2_{\mathrm{T}}$ and for every homeomorphism f of $\mathcal{E}^2_{\mathrm{T}}$ such that S is Jordan holds $f^\circ S$ is Jordan.

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