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Stability of the 4-2 Binary Addition Circuit Cells. Part I

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Summary. To evaluate our formal verification method on a real-size calculation circuit, in this article, we continue to formalize the concept of the 4-2 Binary Addition Cell primitives (FTAs) to define the structures of calculation units for a very fast multiplication algorithm for VLSI implementation [11]. We define the circuit structure of four-types FTAs, TYPE-0 to TYPE-3, using the series constructions of the Generalized Full Adder Circuits (GFAs) that generalized adder to have for each positive and negative weights to inputs and outputs [15]. We then successfully prove its circuit stability of the calculation outputs after four-steps. The motivation for this research is to establish a technique based on formalized mathematics and its applications for calculation circuits with high reliability.

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The terminology and notation used in this paper are introduced in the following papers: [8], [10], [14], [3], [13], [1], [7], [9], [6], [5], [4], [2], [12], and [15]. For simplicity the following abbreviations are introduced

$$\text{BitGFA}i\text{Str} \longmapsto \Sigma_i$$
$$\text{BitGFA}i\text{Circ} \longmapsto \mathfrak{C}_i$$
$$\text{GFA}i\text{AdderOutput} \longmapsto \mathfrak{a}_i$$
$$\text{GFA}i\text{CarryOutput} \longmapsto \mathfrak{c}_i$$
$$\text{InnerVertices} \longmapsto \mathcal{IV}$$

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1. Stability of 4-2 Binary Addition Circuit Cell (TYPE-0)

Let a_1, b_1, c_1, d_1, c_2 be sets. The functor BitFTA0Str(a_1, b_1, c_1, d_1, c_2) yielding an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates is defined by:

(Def. 1) BitFTA0Str(a_1, b_1, c_1, d_1, c_2) $= \Sigma_0(a_1, b_1, c_1) +\cdot \Sigma_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)$.

Let a_1, b_1, c_1, d_1, c_2 be sets. The functor BitFTA0Circ(a_1, b_1, c_1, d_1, c_2) yields a strict Boolean circuit of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) with denotation held in gates and is defined as follows:

(Def. 2) BitFTA0Circ(a_1, b_1, c_1, d_1, c_2) $= \mathfrak{C}_0(a_1, b_1, c_1) +\cdot \mathfrak{C}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)$.

One can prove the following propositions:

(1) Let a_1, b_1, c_1, d_1, c_2 be sets. Then $\mathcal{IV}$(BitFTA0Str(a_1, b_1, c_1, d_1, c_2)) $= \{\langle\langle a_1, b_1\rangle, \mathrm{xor}_2\rangle, \mathfrak{a}_0(a_1, b_1, c_1)\} \cup \{\langle\langle a_1, b_1\rangle, \mathrm{and}_2\rangle, \langle\langle b_1, c_1\rangle, \mathrm{and}_2\rangle, \langle\langle c_1, a_1\rangle, \mathrm{and}_2\rangle, \mathfrak{c}_0(a_1, b_1, c_1)\} \cup \{\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{xor}_2\rangle, \mathfrak{a}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)\} \cup \{\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{and}_2\rangle, \langle\langle c_2, d_1\rangle, \mathrm{and}_2\rangle, \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle, \mathfrak{c}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)\}$.

(2) For all sets a_1, b_1, c_1, d_1, c_2 holds $\mathcal{IV}$(BitFTA0Str(a_1, b_1, c_1, d_1, c_2)) is a binary relation.

(3) For all non pair sets a_1, b_1, c_1, d_1 and for every set c_2 such that $c_2 \neq \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_0(a_1, b_1, c_1))$ holds InputVertices(BitFTA0Str(a_1, b_1, c_1, d_1, c_2)) $= \{a_1, b_1, c_1, d_1, c_2\}$.

(4) Let a_1, b_1, c_1, d_1, c_2 be sets. Then $a_1 \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $b_1 \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $c_1 \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $d_1 \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $c_2 \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle a_1, b_1\rangle, \mathrm{xor}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\mathfrak{a}_0(a_1, b_1, c_1) \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle a_1, b_1\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle b_1, c_1\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle c_1, a_1\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\mathfrak{c}_0(a_1, b_1, c_1) \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{xor}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\mathfrak{a}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1) \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle c_2, d_1\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2) and $\mathfrak{c}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1) \in$ the carrier of BitFTA0Str(a_1, b_1, c_1, d_1, c_2).

(5) Let a_1, b_1, c_1, d_1, c_2 be sets. Then $\langle\langle a_1, b_1\rangle, \mathrm{xor}_2\rangle \in \mathcal{IV}$(BitFTA0Str($a_1$, b_1, c_1, d_1, c_2)) and $\mathfrak{a}_0(a_1, b_1, c_1) \in \mathcal{IV}$(BitFTA0Str($a_1$, b_1, c_1, d_1, c_2)) and

$\langle\langle a_1, b_1\rangle, \mathrm{and}_2\rangle$, $\langle\langle b_1, c_1\rangle, \mathrm{and}_2\rangle$, $\langle\langle c_1, a_1\rangle, \mathrm{and}_2\rangle \in \mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ and $\mathfrak{c}_0(a_1, b_1, c_1) \in \mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ and $\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{xor}_2\rangle$, $\mathfrak{a}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)$, $\langle\langle \mathfrak{a}_0(a_1, b_1, c_1), c_2\rangle, \mathrm{and}_2\rangle$, $\langle\langle c_2, d_1\rangle, \mathrm{and}_2\rangle$, $\langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$, $\mathfrak{c}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1) \in \mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$.

(6) Let a_1, b_1, c_1, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_0(a_1, b_1, c_1))$. Then a_1, b_1, c_1, d_1, $c_2 \in \mathrm{InputVertices}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$.

Let a_1, b_1, c_1, d_1, c_2 be sets. The functor $\mathrm{BitFTA0CarryOutput}(a_1, b_1, c_1, d_1, c_2)$ yields an element of $\mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ and is defined as follows:

(Def. 3) $\mathrm{BitFTA0CarryOutput}(a_1, b_1, c_1, d_1, c_2) = \mathfrak{c}_0(a_1, b_1, c_1)$.

The functor $\mathrm{BitFTA0AdderOutputI}(a_1, b_1, c_1, d_1, c_2)$ yields an element of $\mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ and is defined as follows:

(Def. 4) $\mathrm{BitFTA0AdderOutputI}(a_1, b_1, c_1, d_1, c_2) = \mathfrak{a}_0(a_1, b_1, c_1)$.

The functor $\mathrm{BitFTA0AdderOutputP}(a_1, b_1, c_1, d_1, c_2)$ yielding an element of $\mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ is defined by:

(Def. 5) $\mathrm{BitFTA0AdderOutputP}(a_1, b_1, c_1, d_1, c_2) = \mathfrak{c}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)$.

The functor $\mathrm{BitFTA0AdderOutputQ}(a_1, b_1, c_1, d_1, c_2)$ yields an element of $\mathcal{IV}(\mathrm{BitFTA0Str}(a_1, b_1, c_1, d_1, c_2))$ and is defined by:

(Def. 6) $\mathrm{BitFTA0AdderOutputQ}(a_1, b_1, c_1, d_1, c_2) = \mathfrak{a}_0(\mathfrak{a}_0(a_1, b_1, c_1), c_2, d_1)$.

The following propositions are true:

(7) Let a_1, b_1, c_1 be non pair sets, d_1, c_2 be sets, s be a state of $\mathrm{BitFTA0Circ}(a_1, b_1, c_1, d_1, c_2)$, and a_2, a_3, a_4 be elements of *Boolean*. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_1)$ and $a_4 = s(c_1)$. Then $(\mathrm{Following}(s, 2))(\mathrm{BitFTA0CarryOutput}(a_1, b_1, c_1, d_1, c_2)) = a_2 \wedge a_3 \vee a_3 \wedge a_4 \vee a_4 \wedge a_2$ and $(\mathrm{Following}(s, 2))(\mathrm{BitFTA0AdderOutputI}(a_1, b_1, c_1, d_1, c_2)) = a_2 \oplus a_3 \oplus a_4$.

(8) Let a_1, b_1, c_1, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_0(a_1, b_1, c_1))$. Let s be a state of $\mathrm{BitFTA0Circ}(a_1, b_1, c_1, d_1, c_2)$ and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_1)$ and $a_4 = s(c_1)$ and $a_5 = s(d_1)$ and $a_6 = s(c_2)$. Then $(\mathrm{Following}(s, 2))(\mathfrak{a}_0(a_1, b_1, c_1)) = a_2 \oplus a_3 \oplus a_4$ and $(\mathrm{Following}(s, 2))(a_1) = a_2$ and $(\mathrm{Following}(s, 2))(b_1) = a_3$ and $(\mathrm{Following}(s, 2))(c_1) = a_4$ and $(\mathrm{Following}(s, 2))(d_1) = a_5$ and $(\mathrm{Following}(s, 2))(c_2) = a_6$.

(9) Let a_1, b_1, c_1, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_0(a_1, b_1, c_1))$. Let s be a state of $\mathrm{BitFTA0Circ}(a_1, b_1, c_1, d_1, c_2)$ and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_1)$ and $a_4 = s(c_1)$ and $a_5 = s(d_1)$

and $a_6 = s(c_2)$. Then (Following$(s, 4)$)(BitFTA0AdderOutputP$(a_1, b_1, c_1, d_1, c_2)$) $= (a_2 \oplus a_3 \oplus a_4) \wedge a_6 \vee a_6 \wedge a_5 \vee a_5 \wedge (a_2 \oplus a_3 \oplus a_4)$ and (Following$(s, 4)$)(BitFTA0AdderOutputQ$(a_1, b_1, c_1, d_1, c_2)$) $= a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6$.

(10) Let a_1, b_1, c_1, d_1 be non pair sets and c_2 be a set. If $c_2 \neq \langle\langle d_1, \mathfrak{a}_0(a_1, b_1, c_1)\rangle, \mathrm{and}_2\rangle$, then for every state s of BitFTA0Circ$(a_1, b_1, c_1, d_1, c_2)$ holds Following$(s, 4)$ is stable.

2. Stability of 4-2 Binary Addition Circuit Cell (TYPE-1)

Let a_1, b_2, c_1, d_2, c_2 be sets. The functor BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ yielding an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates is defined by:

(Def. 7) BitFTA1Str$(a_1, b_2, c_1, d_2, c_2) = \Sigma_1(a_1, b_2, c_1) +\cdot \Sigma_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)$.

Let a_1, b_2, c_1, d_2, c_2 be sets. The functor BitFTA1Circ$(a_1, b_2, c_1, d_2, c_2)$ yields a strict Boolean circuit of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ with denotation held in gates and is defined by:

(Def. 8) BitFTA1Circ$(a_1, b_2, c_1, d_2, c_2) = \mathfrak{C}_1(a_1, b_2, c_1) +\cdot \mathfrak{C}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)$.

Next we state several propositions:

(11) Let a_1, b_2, c_1, d_2, c_2 be sets. Then $\mathcal{IV}$(BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$) $= \{\langle\langle a_1, b_2\rangle, \mathrm{xor2c}\rangle, \mathfrak{a}_1(a_1, b_2, c_1)\} \cup \{\langle\langle a_1, b_2\rangle, \mathrm{and2c}\rangle, \langle\langle b_2, c_1\rangle, \mathrm{and}_{2a}\rangle, \langle\langle c_1, a_1\rangle, \mathrm{and}_2\rangle, \mathfrak{c}_1(a_1, b_2, c_1)\} \cup \{\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle, \mathrm{xor2c}\rangle, \mathfrak{a}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)\} \cup \{\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle, \mathrm{and}_{2a}\rangle, \langle\langle c_2, d_2\rangle, \mathrm{and2c}\rangle, \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle, \mathrm{and}_{2b}\rangle, \mathfrak{c}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)\}$.

(12) For all sets a_1, b_2, c_1, d_2, c_2 holds $\mathcal{IV}$(BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$) is a binary relation.

(13) For all non pair sets a_1, b_2, c_1, d_2 and for every set c_2 such that $c_2 \neq \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle, \mathrm{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_1(a_1, b_2, c_1))$ holds InputVertices(BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$) $= \{a_1, b_2, c_1, d_2, c_2\}$.

(14) Let a_1, b_2, c_1, d_2, c_2 be sets. Then $a_1 \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $b_2 \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $c_1 \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $d_2 \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $c_2 \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\langle\langle a_1, b_2\rangle, \mathrm{xor2c}\rangle \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\mathfrak{a}_1(a_1, b_2, c_1) \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\langle\langle a_1, b_2\rangle, \mathrm{and2c}\rangle \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\langle\langle b_2, c_1\rangle, \mathrm{and}_{2a}\rangle \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\langle\langle c_1, a_1\rangle, \mathrm{and}_2\rangle \in$ the carrier of BitFTA1Str$(a_1, b_2, c_1, d_2, c_2)$ and $\mathfrak{c}_1(a_1, b_2,$

$c_1) \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle$, xor2c $\rangle \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\mathfrak{a}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2) \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle$, $\text{and}_{2a} \rangle \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\langle\langle c_2, d_2\rangle$, and2c $\rangle \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle$, $\text{and}_{2b} \rangle \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2) and $\mathfrak{c}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2) \in$ the carrier of BitFTA1Str(a_1, b_2, c_1, d_2, c_2).

(15) Let a_1, b_2, c_1, d_2, c_2 be sets. Then $\langle\langle a_1, b_2\rangle$, xor2c $\rangle \in \mathcal{IV}$(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)) and $\mathfrak{a}_1(a_1, b_2, c_1) \in \mathcal{IV}$(BitFTA1Str($a_1$, b_2, c_1, d_2, c_2)) and $\langle\langle a_1, b_2\rangle$, and2c $\rangle$, $\langle\langle b_2, c_1\rangle$, $\text{and}_{2a} \rangle$, $\langle\langle c_1, a_1\rangle$, $\text{and}_2 \rangle \in \mathcal{IV}$(BitFTA1Str($a_1$, b_2, c_1, d_2, c_2)) and $\mathfrak{c}_1(a_1, b_2, c_1) \in \mathcal{IV}$(BitFTA1Str($a_1$, b_2, c_1, d_2, c_2)) and $\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle$, xor2c $\rangle$, $\mathfrak{a}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)$, $\langle\langle \mathfrak{a}_1(a_1, b_2, c_1), c_2\rangle$, $\text{and}_{2a} \rangle$, $\langle\langle c_2, d_2\rangle$, and2c $\rangle$, $\langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle$, $\text{and}_{2b} \rangle$, $\mathfrak{c}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2) \in \mathcal{IV}$(BitFTA1Str($a_1$, b_2, c_1, d_2, c_2)).

(16) Let a_1, b_2, c_1, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle$, $\text{and}_{2b} \rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_1(a_1, b_2, c_1))$. Then a_1, b_2, c_1, d_2, $c_2 \in$ InputVertices(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)).

Let a_1, b_2, c_1, d_2, c_2 be sets. The functor BitFTA1CarryOutput(a_1, b_2, c_1, d_2, c_2) yielding an element of $\mathcal{IV}$(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)) is defined as follows:

(Def. 9) BitFTA1CarryOutput(a_1, b_2, c_1, d_2, c_2) $= \mathfrak{c}_1(a_1, b_2, c_1)$.

The functor BitFTA1AdderOutputI(a_1, b_2, c_1, d_2, c_2) yields an element of $\mathcal{IV}$(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)) and is defined by:

(Def. 10) BitFTA1AdderOutputI(a_1, b_2, c_1, d_2, c_2) $= \mathfrak{a}_1(a_1, b_2, c_1)$.

The functor BitFTA1AdderOutputP(a_1, b_2, c_1, d_2, c_2) yields an element of $\mathcal{IV}$(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)) and is defined as follows:

(Def. 11) BitFTA1AdderOutputP(a_1, b_2, c_1, d_2, c_2) $= \mathfrak{c}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)$.

The functor BitFTA1AdderOutputQ(a_1, b_2, c_1, d_2, c_2) yielding an element of $\mathcal{IV}$(BitFTA1Str(a_1, b_2, c_1, d_2, c_2)) is defined as follows:

(Def. 12) BitFTA1AdderOutputQ(a_1, b_2, c_1, d_2, c_2) $= \mathfrak{a}_2(\mathfrak{a}_1(a_1, b_2, c_1), c_2, d_2)$.

The following four propositions are true:

(17) Let a_1, b_2, c_1 be non pair sets, d_2, c_2 be sets, s be a state of BitFTA1Circ(a_1, b_2, c_1, d_2, c_2), and a_2, a_3, a_4 be elements of *Boolean*. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_2)$ and $a_4 = s(c_1)$. Then (Following($s, 2$))(BitFTA1CarryOutput(a_1, b_2, c_1, d_2, c_2)) $= a_2 \wedge \neg a_3 \vee \neg a_3 \wedge a_4 \vee a_4 \wedge a_2$ and (Following($s, 2$))(BitFTA1AdderOutputI(a_1, b_2, c_1, d_2, c_2)) $= \neg(a_2 \oplus \neg a_3 \oplus a_4)$.

(18) Let a_1, b_2, c_1, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle$, $\text{and}_{2b} \rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_1(a_1, b_2, c_1))$. Let s be a state of BitFTA1Circ(a_1, b_2, c_1, d_2, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of

Boolean. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_2)$ and $a_4 = s(c_1)$ and $a_5 = s(d_2)$ and $a_6 = s(c_2)$. Then $(\mathrm{Following}(s,2))(\mathfrak{a}_1(a_1, b_2, c_1)) = \neg(a_2 \oplus \neg a_3 \oplus a_4)$ and $(\mathrm{Following}(s,2))(a_1) = a_2$ and $(\mathrm{Following}(s,2))(b_2) = a_3$ and $(\mathrm{Following}(s,2))(c_1) = a_4$ and $(\mathrm{Following}(s,2))(d_2) = a_5$ and $(\mathrm{Following}(s,2))(c_2) = a_6$.

(19) Let a_1, b_2, c_1, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle, \mathrm{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_1(a_1, b_2, c_1))$. Let s be a state of BitFTA1Circ(a_1, b_2, c_1, d_2, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_1)$ and $a_3 = s(b_2)$ and $a_4 = s(c_1)$ and $a_5 = s(d_2)$ and $a_6 = s(c_2)$. Then $(\mathrm{Following}(s,4))(\mathrm{BitFTA1AdderOutputP}(a_1, b_2, c_1, d_2, c_2)) = \neg((a_2 \oplus \neg a_3 \oplus a_4) \wedge a_6 \vee a_6 \wedge \neg a_5 \vee \neg a_5 \wedge (a_2 \oplus \neg a_3 \oplus a_4))$ and $(\mathrm{Following}(s,4))(\mathrm{BitFTA1AdderOutputQ}(a_1, b_2, c_1, d_2, c_2)) = a_2 \oplus \neg a_3 \oplus a_4 \oplus \neg a_5 \oplus a_6$.

(20) Let a_1, b_2, c_1, d_2 be non pair sets and c_2 be a set. If $c_2 \neq \langle\langle d_2, \mathfrak{a}_1(a_1, b_2, c_1)\rangle, \mathrm{and}_{2b}\rangle$, then for every state s of BitFTA1Circ(a_1, b_2, c_1, d_2, c_2) holds Following$(s, 4)$ is stable.

3. Stability of 4-2 Binary Addition Circuit Cell (TYPE-2)

Let a_7, b_1, c_3, d_1, c_2 be sets. The functor BitFTA2Str(a_7, b_1, c_3, d_1, c_2) yielding an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates is defined by:

(Def. 13) $\mathrm{BitFTA2Str}(a_7, b_1, c_3, d_1, c_2) = \Sigma_2(a_7, b_1, c_3) +\cdot \Sigma_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)$.

Let a_7, b_1, c_3, d_1, c_2 be sets. The functor BitFTA2Circ(a_7, b_1, c_3, d_1, c_2) yielding a strict Boolean circuit of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) with denotation held in gates is defined by:

(Def. 14) $\mathrm{BitFTA2Circ}(a_7, b_1, c_3, d_1, c_2) = \mathfrak{C}_2(a_7, b_1, c_3) +\cdot \mathfrak{C}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)$.

Next we state several propositions:

(21) Let a_7, b_1, c_3, d_1, c_2 be sets. Then $\mathcal{IV}(\mathrm{BitFTA2Str}(a_7, b_1, c_3, d_1, c_2)) = \{\langle\langle a_7, b_1\rangle, \mathrm{xor2c}\rangle, \mathfrak{a}_2(a_7, b_1, c_3)\} \cup \{\langle\langle a_7, b_1\rangle, \mathrm{and}_{2a}\rangle, \langle\langle b_1, c_3\rangle, \mathrm{and2c}\rangle, \langle\langle c_3, a_7\rangle, \mathrm{and}_{2b}\rangle, \mathfrak{c}_2(a_7, b_1, c_3)\} \cup \{\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \mathrm{xor2c}\rangle, \mathfrak{a}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)\} \cup \{\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \mathrm{and2c}\rangle, \langle\langle c_2, d_1\rangle, \mathrm{and}_{2a}\rangle, \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \mathrm{and}_2\rangle, \mathfrak{c}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)\}$.

(22) For all sets a_7, b_1, c_3, d_1, c_2 holds $\mathcal{IV}(\mathrm{BitFTA2Str}(a_7, b_1, c_3, d_1, c_2))$ is a binary relation.

(23) For all non pair sets a_7, b_1, c_3, d_1 and for every set c_2 such that $c_2 \neq \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_2(a_7, b_1, c_3))$ holds $\mathrm{InputVertices}(\mathrm{BitFTA2Str}(a_7, b_1, c_3, d_1, c_2)) = \{a_7, b_1, c_3, d_1, c_2\}$.

(24) Let a_7, b_1, c_3, d_1, c_2 be sets. Then $a_7 \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $b_1 \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $c_3 \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $d_1 \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $c_2 \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle a_7, b_1\rangle, \text{xor2c}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\mathfrak{a}_2(a_7, b_1, c_3) \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle a_7, b_1\rangle, \text{and}_{2a}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle b_1, c_3\rangle, \text{and2c}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle c_3, a_7\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\mathfrak{c}_2(a_7, b_1, c_3) \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \text{xor2c}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\mathfrak{a}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1) \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \text{and2c}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle c_2, d_1\rangle, \text{and}_{2a}\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \text{and}_2\rangle \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2) and $\mathfrak{c}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1) \in$ the carrier of BitFTA2Str(a_7, b_1, c_3, d_1, c_2).

(25) Let a_7, b_1, c_3, d_1, c_2 be sets. Then $\langle\langle a_7, b_1\rangle, \text{xor2c}\rangle \in \mathcal{IV}$(BitFTA2Str($a_7$, b_1, c_3, d_1, c_2)) and $\mathfrak{a}_2(a_7, b_1, c_3) \in \mathcal{IV}$(BitFTA2Str($a_7$, b_1, c_3, d_1, c_2)) and $\langle\langle a_7, b_1\rangle, \text{and}_{2a}\rangle$, $\langle\langle b_1, c_3\rangle, \text{and2c}\rangle$, $\langle\langle c_3, a_7\rangle, \text{and}_{2b}\rangle \in \mathcal{IV}$(BitFTA2Str($a_7$, b_1, c_3, d_1, c_2)) and $\mathfrak{c}_2(a_7, b_1, c_3) \in \mathcal{IV}$(BitFTA2Str($a_7$, b_1, c_3, d_1, c_2)) and $\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \text{xor2c}\rangle$, $\mathfrak{a}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)$, $\langle\langle \mathfrak{a}_2(a_7, b_1, c_3), c_2\rangle, \text{and2c}\rangle$, $\langle\langle c_2, d_1\rangle, \text{and}_{2a}\rangle$, $\langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \text{and}_2\rangle$, $\mathfrak{c}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1) \in \mathcal{IV}$(BitFTA2Str($a_7$, b_1, c_3, d_1, c_2)).

(26) Let a_7, b_1, c_3, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \text{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_2(a_7, b_1, c_3))$. Then a_7, b_1, c_3, d_1, $c_2 \in$ InputVertices(BitFTA2Str(a_7, b_1, c_3, d_1, c_2)).

Let a_7, b_1, c_3, d_1, c_2 be sets. The functor BitFTA2CarryOutput(a_7, b_1, c_3, d_1, c_2) yields an element of $\mathcal{IV}$(BitFTA2Str(a_7, b_1, c_3, d_1, c_2)) and is defined as follows:

(Def. 15) BitFTA2CarryOutput(a_7, b_1, c_3, d_1, c_2) $= \mathfrak{c}_2(a_7, b_1, c_3)$.

The functor BitFTA2AdderOutputI(a_7, b_1, c_3, d_1, c_2) yields an element of $\mathcal{IV}$(BitFTA2Str(a_7, b_1, c_3, d_1, c_2)) and is defined as follows:

(Def. 16) BitFTA2AdderOutputI(a_7, b_1, c_3, d_1, c_2) $= \mathfrak{a}_2(a_7, b_1, c_3)$.

The functor BitFTA2AdderOutputP(a_7, b_1, c_3, d_1, c_2) yields an element of $\mathcal{IV}$(BitFTA2Str(a_7, b_1, c_3, d_1, c_2)) and is defined by:

(Def. 17) BitFTA2AdderOutputP(a_7, b_1, c_3, d_1, c_2) $= \mathfrak{c}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)$.

The functor BitFTA2AdderOutputQ(a_7, b_1, c_3, d_1, c_2) yielding an element of $\mathcal{IV}$(BitFTA2Str(a_7, b_1, c_3, d_1, c_2)) is defined as follows:

(Def. 18) BitFTA2AdderOutputQ(a_7, b_1, c_3, d_1, c_2) $= \mathfrak{a}_1(\mathfrak{a}_2(a_7, b_1, c_3), c_2, d_1)$.

One can prove the following propositions:

(27) Let a_7, b_1, c_3 be non pair sets, d_1, c_2 be sets, s be a state of BitFTA2Circ(a_7, b_1, c_3, d_1, c_2), and a_2, a_3, a_4 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_1)$ and $a_4 = s(c_3)$. Then (Following$(s, 2)$)(BitFTA2CarryOutput(a_7, b_1, c_3, d_1, c_2)) $= \neg(\neg a_2 \wedge a_3 \vee a_3 \wedge \neg a_4 \vee \neg a_4 \wedge \neg a_2)$ and (Following$(s, 2)$)(BitFTA2AdderOutputI(a_7, b_1, c_3, d_1, c_2)) $= \neg a_2 \oplus a_3 \oplus \neg a_4$.

(28) Let a_7, b_1, c_3, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_2(a_7, b_1, c_3))$. Let s be a state of BitFTA2Circ(a_7, b_1, c_3, d_1, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_1)$ and $a_4 = s(c_3)$ and $a_5 = s(d_1)$ and $a_6 = s(c_2)$. Then (Following$(s, 2)$)($\mathfrak{a}_2(a_7, b_1, c_3)$) $= \neg a_2 \oplus a_3 \oplus \neg a_4$ and (Following$(s, 2)$)$(a_7) = a_2$ and (Following$(s, 2)$)$(b_1) = a_3$ and (Following$(s, 2)$)$(c_3) = a_4$ and (Following$(s, 2)$)$(d_1) = a_5$ and (Following$(s, 2)$)$(c_2) = a_6$.

(29) Let a_7, b_1, c_3, d_1 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \mathrm{and}_2\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_2(a_7, b_1, c_3))$. Let s be a state of BitFTA2Circ(a_7, b_1, c_3, d_1, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_1)$ and $a_4 = s(c_3)$ and $a_5 = s(d_1)$ and $a_6 = s(c_2)$. Then (Following$(s, 4)$)(BitFTA2AdderOutputP(a_7, b_1, c_3, d_1, c_2)) $= (\neg a_2 \oplus a_3 \oplus \neg a_4) \wedge \neg a_6 \vee \neg a_6 \wedge a_5 \vee a_5 \wedge (\neg a_2 \oplus a_3 \oplus \neg a_4)$ and (Following$(s, 4)$)(BitFTA2AdderOutputQ(a_7, b_1, c_3, d_1, c_2)) $= \neg(\neg a_2 \oplus a_3 \oplus \neg a_4 \oplus a_5 \oplus \neg a_6)$.

(30) Let a_7, b_1, c_3, d_1 be non pair sets and c_2 be a set. If $c_2 \neq \langle\langle d_1, \mathfrak{a}_2(a_7, b_1, c_3)\rangle, \mathrm{and}_2\rangle$, then for every state s of BitFTA2Circ(a_7, b_1, c_3, d_1, c_2) holds Following$(s, 4)$ is stable.

4. Stability of 4-2 Binary Addition Circuit Cell (TYPE-3)

Let a_7, b_2, c_3, d_2, c_2 be sets. The functor BitFTA3Str(a_7, b_2, c_3, d_2, c_2) yields an unsplit non void strict non empty many sorted signature with arity held in gates and Boolean denotation held in gates and is defined by:

(Def. 19) BitFTA3Str(a_7, b_2, c_3, d_2, c_2) $= \Sigma_3(a_7, b_2, c_3) +\cdot \Sigma_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)$.

Let a_7, b_2, c_3, d_2, c_2 be sets. The functor BitFTA3Circ(a_7, b_2, c_3, d_2, c_2) yielding a strict Boolean circuit of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) with denotation held in gates is defined by:

(Def. 20) BitFTA3Circ(a_7, b_2, c_3, d_2, c_2) $= \mathfrak{C}_3(a_7, b_2, c_3) +\cdot \mathfrak{C}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)$.

We now state several propositions:

(31) Let a_7, b_2, c_3, d_2, c_2 be sets. Then $\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) = $\{\langle\langle a_7, b_2\rangle, \text{xor}_2\rangle, \mathfrak{a}_3(a_7, b_2, c_3)\} \cup \{\langle\langle a_7, b_2\rangle, \text{and}_{2b}\rangle, \langle\langle b_2, c_3\rangle, \text{and}_{2b}\rangle, \langle\langle c_3, a_7\rangle, \text{and}_{2b}\rangle, \mathfrak{c}_3(a_7, b_2, c_3)\} \cup \{\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{xor}_2\rangle, \mathfrak{a}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)\} \cup \{\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{and}_{2b}\rangle, \langle\langle c_2, d_2\rangle, \text{and}_{2b}\rangle, \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle, \mathfrak{c}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)\}$.

(32) For all sets a_7, b_2, c_3, d_2, c_2 holds $\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) is a binary relation.

(33) For all non pair sets a_7, b_2, c_3, d_2 and for every set c_2 such that $c_2 \neq \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_3(a_7, b_2, c_3))$ holds InputVertices(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) = $\{a_7, b_2, c_3, d_2, c_2\}$.

(34) Let a_7, b_2, c_3, d_2, c_2 be sets. Then $a_7 \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $b_2 \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $c_3 \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $d_2 \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $c_2 \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle a_7, b_2\rangle, \text{xor}_2\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\mathfrak{a}_3(a_7, b_2, c_3) \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle a_7, b_2\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle b_2, c_3\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle c_3, a_7\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\mathfrak{c}_3(a_7, b_2, c_3) \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{xor}_2\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\mathfrak{a}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2) \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle c_2, d_2\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2) and $\mathfrak{c}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2) \in$ the carrier of BitFTA3Str(a_7, b_2, c_3, d_2, c_2).

(35) Let a_7, b_2, c_3, d_2, c_2 be sets. Then $\langle\langle a_7, b_2\rangle, \text{xor}_2\rangle \in \mathcal{IV}$(BitFTA3Str($a_7$, b_2, c_3, d_2, c_2)) and $\mathfrak{a}_3(a_7, b_2, c_3) \in \mathcal{IV}$(BitFTA3Str($a_7$, b_2, c_3, d_2, c_2)) and $\langle\langle a_7, b_2\rangle, \text{and}_{2b}\rangle, \langle\langle b_2, c_3\rangle, \text{and}_{2b}\rangle, \langle\langle c_3, a_7\rangle, \text{and}_{2b}\rangle \in \mathcal{IV}$(BitFTA3Str($a_7$, b_2, c_3, d_2, c_2)) and $\mathfrak{c}_3(a_7, b_2, c_3) \in \mathcal{IV}$(BitFTA3Str($a_7$, b_2, c_3, d_2, c_2)) and $\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{xor}_2\rangle$, $\mathfrak{a}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)$, $\langle\langle \mathfrak{a}_3(a_7, b_2, c_3), c_2\rangle, \text{and}_{2b}\rangle$, $\langle\langle c_2, d_2\rangle, \text{and}_{2b}\rangle$, $\langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$, $\mathfrak{c}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2) \in \mathcal{IV}$(BitFTA3Str($a_7$, b_2, c_3, d_2, c_2)).

(36) Let a_7, b_2, c_3, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_3(a_7, b_2, c_3))$. Then a_7, b_2, c_3, d_2, $c_2 \in$ InputVertices(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)).

Let a_7, b_2, c_3, d_2, c_2 be sets. The functor BitFTA3CarryOutput(a_7, b_2, c_3, d_2, c_2) yields an element of $\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) and is defined by:

(Def. 21) BitFTA3CarryOutput(a_7, b_2, c_3, d_2, c_2) = $\mathfrak{c}_3(a_7, b_2, c_3)$.

The functor BitFTA3AdderOutputI(a_7, b_2, c_3, d_2, c_2) yields an element of

$\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) and is defined by:

(Def. 22) BitFTA3AdderOutputI(a_7, b_2, c_3, d_2, c_2) $= \mathfrak{a}_3(a_7, b_2, c_3)$.

The functor BitFTA3AdderOutputP(a_7, b_2, c_3, d_2, c_2) yields an element of $\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) and is defined by:

(Def. 23) BitFTA3AdderOutputP(a_7, b_2, c_3, d_2, c_2) $= \mathfrak{c}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)$.

The functor BitFTA3AdderOutputQ(a_7, b_2, c_3, d_2, c_2) yielding an element of $\mathcal{IV}$(BitFTA3Str(a_7, b_2, c_3, d_2, c_2)) is defined by:

(Def. 24) BitFTA3AdderOutputQ(a_7, b_2, c_3, d_2, c_2) $= \mathfrak{a}_3(\mathfrak{a}_3(a_7, b_2, c_3), c_2, d_2)$.

One can prove the following propositions:

(37) Let a_7, b_2, c_3 be non pair sets, d_2, c_2 be sets, s be a state of BitFTA3Circ(a_7, b_2, c_3, d_2, c_2), and a_2, a_3, a_4 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_2)$ and $a_4 = s(c_3)$. Then (Following($s, 2$))(BitFTA3CarryOutput(a_7, b_2, c_3, d_2, c_2)) $= \neg(\neg a_2 \wedge \neg a_3 \vee \neg a_3 \wedge \neg a_4 \vee \neg a_4 \wedge \neg a_2)$ and (Following($s, 2$))(BitFTA3AdderOutputI(a_7, b_2, c_3, d_2, c_2)) $= \neg(\neg a_2 \oplus \neg a_3 \oplus \neg a_4)$.

(38) Let a_7, b_2, c_3, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_3(a_7, b_2, c_3))$. Let s be a state of BitFTA3Circ(a_7, b_2, c_3, d_2, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_2)$ and $a_4 = s(c_3)$ and $a_5 = s(d_2)$ and $a_6 = s(c_2)$. Then (Following($s, 2$))($\mathfrak{a}_3(a_7, b_2, c_3)$) $= \neg(\neg a_2 \oplus \neg a_3 \oplus \neg a_4)$ and (Following($s, 2$))(a_7) $= a_2$ and (Following($s, 2$))(b_2) $= a_3$ and (Following($s, 2$))(c_3) $= a_4$ and (Following($s, 2$))(d_2) $= a_5$ and (Following($s, 2$))(c_2) $= a_6$.

(39) Let a_7, b_2, c_3, d_2 be non pair sets and c_2 be a set. Suppose $c_2 \neq \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$ and $c_2 \notin \mathcal{IV}(\Sigma_3(a_7, b_2, c_3))$. Let s be a state of BitFTA3Circ(a_7, b_2, c_3, d_2, c_2) and a_2, a_3, a_4, a_5, a_6 be elements of *Boolean*. Suppose $a_2 = s(a_7)$ and $a_3 = s(b_2)$ and $a_4 = s(c_3)$ and $a_5 = s(d_2)$ and $a_6 = s(c_2)$. Then (Following($s, 4$))(BitFTA3AdderOutputP(a_7, b_2, c_3, d_2, c_2)) $= \neg((\neg a_2 \oplus \neg a_3 \oplus \neg a_4) \wedge \neg a_6 \vee \neg a_6 \wedge \neg a_5 \vee \neg a_5 \wedge (\neg a_2 \oplus \neg a_3 \oplus \neg a_4))$ and (Following($s, 4$))(BitFTA3AdderOutputQ(a_7, b_2, c_3, d_2, c_2)) $= \neg(\neg a_2 \oplus \neg a_3 \oplus \neg a_4 \oplus \neg a_5 \oplus \neg a_6)$.

(40) Let a_7, b_2, c_3, d_2 be non pair sets and c_2 be a set. If $c_2 \neq \langle\langle d_2, \mathfrak{a}_3(a_7, b_2, c_3)\rangle, \text{and}_{2b}\rangle$, then for every state s of BitFTA3Circ(a_7, b_2, c_3, d_2, c_2) holds Following($s, 4$) is stable.

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