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Basic Properties of Periodic Functions

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Summary. In this article we present definitions, basic properties and some examples of periodic functions according to [5].

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The papers [2], [6], [3], [10], [11], [9], [8], [1], [4], and [7] provide the terminology and notation for this paper.

1. Basic Properties of a Period of a Function

We use the following convention: x, t, t_1, t_2, r, a, b are real numbers and F, G are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$ and let t be a real number. We say that t is a period of F if and only if:

(Def. 1) $t \neq 0$ and for every x holds $x \in \operatorname{dom} F$ iff $x+t \in \operatorname{dom} F$ and if $x \in \operatorname{dom} F$, then $F(x) = F(x+t)$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$. We say that F is periodic if and only if:

(Def. 2) There exists t which is a period of F.

We now state a number of propositions:

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(1) t is a period of F iff $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x + t$, $x - t \in \operatorname{dom} F$ and $F(x) = F(x + t)$.
(2) If t is a period of F and a period of G, then t is a period of $F + G$.
(3) If t is a period of F and a period of G, then t is a period of $F - G$.
(4) If t is a period of F and a period of G, then t is a period of $F\,G$.
(5) If t is a period of F and a period of G, then t is a period of F/G.
(6) If t is a period of F, then t is a period of $-F$.
(7) If t is a period of F, then t is a period of $r\,F$.
(8) If t is a period of F, then t is a period of $r + F$.
(9) If t is a period of F, then t is a period of $F - r$.
(10) If t is a period of F, then t is a period of $|F|$.
(11) If t is a period of F, then t is a period of F^{-1}.
(12) If t is a period of F, then t is a period of $F^{\mathbf{2}}$.
(13) If t is a period of F, then for every x such that $x \in \operatorname{dom} F$ holds $F(x) = F(x - t)$.
(14) If t is a period of F, then $-t$ is a period of F.
(15) If t_1 is a period of F and t_2 is a period of F and $t_1 + t_2 \neq 0$, then $t_1 + t_2$ is a period of F.
(16) If t_1 is a period of F and t_2 is a period of F and $t_1 - t_2 \neq 0$, then $t_1 - t_2$ is a period of F.
(17) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x + t$, $x - t \in \operatorname{dom} F$ and $F(x + t) = F(x - t)$. Then $2 \cdot t$ is a period of F and F is periodic.
(18) Suppose $t_1 + t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x + t_1$, $x - t_1$, $x + t_2$, $x - t_2 \in \operatorname{dom} F$ and $F(x + t_1) = F(x - t_2)$. Then $t_1 + t_2$ is a period of F and F is periodic.
(19) Suppose $t_1 - t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x + t_1$, $x - t_1$, $x + t_2$, $x - t_2 \in \operatorname{dom} F$ and $F(x + t_1) = F(x + t_2)$. Then $t_1 - t_2$ is a period of F and F is periodic.
(20) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x + t$, $x - t \in \operatorname{dom} F$ and $F(x+t) = F(x)^{-1}$. Then $2 \cdot t$ is a period of F and F is periodic.

Let us observe that there exists a partial function from $\mathbb{R}$ to $\mathbb{R}$ which is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check that $-F$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$ and let r be a real number. One can check the following observations:

* $r\,F$ is periodic,
* $r + F$ is periodic, and

* $F - r$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check the following observations:

* $|F|$ is periodic,
* F^{-1} is periodic, and
* $F^{\mathbf{2}}$ is periodic.

2. Some Examples

Let us note that the function sin is periodic and the function cos is periodic.

We now state two propositions:

(21) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function sin.

(22) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function cos.

Let us observe that the function cosec is periodic and the function sec is periodic.

We now state two propositions:

(23) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function sec.

(24) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of the function cosec.

Let us mention that the function tan is periodic and the function cot is periodic.

Next we state a number of propositions:

(25) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of the function tan.

(26) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of the function cot.

(27) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of |the function sin|.

(28) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of |the function cos|.

(29) For every element k of $\mathbb{N}$ holds $\frac{\pi}{2} \cdot (k+1)$ is a period of |the function sin| + |the function cos|.

(30) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function sin)$^{\mathbf{2}}$.

(31) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function cos)$^{\mathbf{2}}$.

(32) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function sin) (the function cos).

(33) For every element k of $\mathbb{N}$ holds $\pi \cdot (k+1)$ is a period of (the function cos) (the function sin).

(34) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of $b + a$ (the function sin).

(35) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of a (the function sin)$-b$.

(36) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of $b + a$ (the function cos).

(37) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of a (the function cos)$-b$.

(38) If $\mathrm{dom}\, F = \mathbb{R}$ and for every real number x holds $F(x) = a$, then for every element k of $\mathbb{N}$ holds $k+1$ is a period of F.

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