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Linear Transformations of Euclidean Topological Spaces. Part II

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Summary. We prove a number of theorems concerning various notions used in the theory of continuity of barycentric coordinates.

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The papers [2], [9], [4], [5], [6], [14], [10], [25], [13], [16], [3], [7], [12], [1], [24], [15], [21], [23], [19], [17], [8], [11], [22], [20], and [18] provide the terminology and notation for this paper.

1. Correspondence Between Euclidean Topological Space and Vector Space over $\mathbb{R}_{\mathrm{F}}$

For simplicity, we follow the rules: X denotes a set, n, m, k denote natural numbers, K denotes a field, f denotes an n-element real-valued finite sequence, and M denotes a matrix over $\mathbb{R}_{\mathrm{F}}$ of dimension $n \times m$.

One can prove the following propositions:

(1) X is a linear combination of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ if and only if X is a linear combination of $\mathcal{E}^n_{\mathrm{T}}$.

(2) Let L_2 be a linear combination of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ and L_1 be a linear combination of $\mathcal{E}^n_{\mathrm{T}}$. If $L_1 = L_2$, then the support of $L_1 =$ the support of L_2.

(3) Let F be a finite sequence of elements of $\mathcal{E}^n_{\mathrm{T}}$, f_1 be a function from $\mathcal{E}^n_{\mathrm{T}}$ into $\mathbb{R}$, F_1 be a finite sequence of elements of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$, and f_2 be a function from the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ into $\mathbb{R}_{\mathrm{F}}$. If $f_1 = f_2$ and $F = F_1$, then $f_1\, F = f_2\, F_1$.

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(4) Let F be a finite sequence of elements of $\mathcal{E}^n_{\mathrm{T}}$ and F_1 be a finite sequence of elements of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$. If $F_1 = F$, then $\sum F = \sum F_1$.

(5) Let L_2 be a linear combination of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ and L_1 be a linear combination of $\mathcal{E}^n_{\mathrm{T}}$. If $L_1 = L_2$, then $\sum L_1 = \sum L_2$.

(6) Let A_2 be a subset of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ and A_1 be a subset of $\mathcal{E}^n_{\mathrm{T}}$. If $A_2 = A_1$, then $\Omega_{\mathrm{Lin}(A_1)} = \Omega_{\mathrm{Lin}(A_2)}$.

(7) Let A_2 be a subset of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ and A_1 be a subset of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $A_2 = A_1$. Then A_2 is linearly independent if and only if A_1 is linearly independent.

(8) Let V be a vector space over K, W be a subspace of V, and L be a linear combination of V. Then $L\restriction$the carrier of W is a linear combination of W.

(9) Let V be a vector space over K, A be a linearly independent subset of V, and L_3, L_4 be linear combinations of V. Suppose the support of $L_3 \subseteq A$ and the support of $L_4 \subseteq A$ and $\sum L_3 = \sum L_4$. Then $L_3 = L_4$.

(10) Let V be a real linear space, W be a subspace of V, and L be a linear combination of V. Then $L\restriction$the carrier of W is a linear combination of W.

(11) Let U be a subspace of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$ and W be a subspace of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $\Omega_U = \Omega_W$. Then X is a linear combination of U if and only if X is a linear combination of W.

(12) Let U be a subspace of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$, W be a subspace of $\mathcal{E}^n_{\mathrm{T}}$, L_5 be a linear combination of U, and L_6 be a linear combination of W. If $L_5 = L_6$, then the support of $L_5 =$ the support of L_6 and $\sum L_5 = \sum L_6$.

Let us consider m, K and let A be a subset of the m-dimension vector space over K. Note that $\mathrm{Lin}(A)$ is finite dimensional.

2. Correspondence Between the Mx2Tran Operator and Decomposition of a Vector in Basis

The following propositions are true:

(13) If $\mathrm{rk}(M) = n$, then M is an ordered basis of $\mathrm{Lin}(\mathrm{lines}(M))$.

(14) Let V, W be vector spaces over K, T be a linear transformation from V to W, A be a subset of V, and L be a linear combination of A. If $T\restriction A$ is one-to-one, then $T(\sum L) = \sum(T^{@}L)$.

(15) Let S be a subset of $\mathrm{Seg}\, n$. Suppose $M\restriction S$ is one-to-one and $\mathrm{rng}(M\restriction S) = \mathrm{lines}(M)$. Then there exists a linear combination L of $\mathrm{lines}(M)$ such that $\sum L = (\mathrm{Mx2Tran}\, M)(f)$ and for every k such that $k \in S$ holds $L(\mathrm{Line}(M,k)) = \sum \mathrm{Seq}(f\restriction M^{-1}(\{\mathrm{Line}(M,k)\}))$.

(16) Suppose M is without repeated line. Then there exists a linear combination L of lines(M) such that $\sum L = (\mathrm{Mx2Tran}\, M)(f)$ and for every k such that $k \in \operatorname{dom} f$ holds $L(\mathrm{Line}(M, k)) = f(k)$.

(17) For every ordered basis B of Lin(lines(M)) such that $B = M$ and for every element M_1 of Lin(lines(M)) such that $M_1 = (\mathrm{Mx2Tran}\, M)(f)$ holds $M_1 \rightarrow B = f$.

(18) $\operatorname{rng} \mathrm{Mx2Tran}\, M = \Omega_{\mathrm{Lin}(\mathrm{lines}(M))}$.

(19) Let F be a one-to-one finite sequence of elements of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $\operatorname{rng} F$ is linearly independent. Then there exists a square matrix M over $\mathbb{R}_{\mathrm{F}}$ of dimension n such that M is invertible and $M \restriction \operatorname{len} F = F$.

(20) Let B be an ordered basis of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$. If $B = \mathrm{MX2FinS}(I^{n \times n}_{\mathbb{R}_{\mathrm{F}}})$, then $f \in \mathrm{Lin}(\operatorname{rng}(B \restriction k))$ iff $f = (f \restriction k) \frown ((n -' k) \mapsto 0)$.

(21) Let F be a one-to-one finite sequence of elements of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $\operatorname{rng} F$ is linearly independent. Let B be an ordered basis of the n-dimension vector space over $\mathbb{R}_{\mathrm{F}}$. Suppose $B = \mathrm{MX2FinS}(I^{n \times n}_{\mathbb{R}_{\mathrm{F}}})$. Let M be a square matrix over $\mathbb{R}_{\mathrm{F}}$ of dimension n. If M is invertible and $M \restriction \operatorname{len} F = F$, then $(\mathrm{Mx2Tran}\, M)^{\circ}(\Omega_{\mathrm{Lin}(\operatorname{rng}(B \restriction \operatorname{len} F))}) = \Omega_{\mathrm{Lin}(\operatorname{rng} F)}$.

(22) Let A, B be linearly independent subsets of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $\overline{\overline{A}} = \overline{\overline{B}}$. Then there exists a square matrix M over $\mathbb{R}_{\mathrm{F}}$ of dimension n such that M is invertible and $(\mathrm{Mx2Tran}\, M)^{\circ}(\Omega_{\mathrm{Lin}(A)}) = \Omega_{\mathrm{Lin}(B)}$.

3. Preservation of Linear and Affine Independence of Vectors by the Mx2Tran Operator

The following propositions are true:

(23) For every linearly independent subset A of $\mathcal{E}^n_{\mathrm{T}}$ such that $\mathrm{rk}(M) = n$ holds $(\mathrm{Mx2Tran}\, M)^{\circ} A$ is linearly independent.

(24) For every affinely independent subset A of $\mathcal{E}^n_{\mathrm{T}}$ such that $\mathrm{rk}(M) = n$ holds $(\mathrm{Mx2Tran}\, M)^{\circ} A$ is affinely independent.

(25) Let A be an affinely independent subset of $\mathcal{E}^n_{\mathrm{T}}$. Suppose $\mathrm{rk}(M) = n$. Let v be an element of $\mathcal{E}^n_{\mathrm{T}}$. If $v \in \mathrm{Affin}\, A$, then $(\mathrm{Mx2Tran}\, M)(v) \in \mathrm{Affin}((\mathrm{Mx2Tran}\, M)^{\circ} A)$ and for every f holds $(v \rightarrow A)(f) = ((\mathrm{Mx2Tran}\, M)(v) \rightarrow (\mathrm{Mx2Tran}\, M)^{\circ} A)((\mathrm{Mx2Tran}\, M)(f))$.

(26) For every linearly independent subset A of $\mathcal{E}^m_{\mathrm{T}}$ such that $\mathrm{rk}(M) = n$ holds $(\mathrm{Mx2Tran}\, M)^{-1}(A)$ is linearly independent.

(27) For every affinely independent subset A of $\mathcal{E}^m_{\mathrm{T}}$ such that $\mathrm{rk}(M) = n$ holds $(\mathrm{Mx2Tran}\, M)^{-1}(A)$ is affinely independent.

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