FORMALIZED MATHEMATICS
Vol. 21, No. 4, Pages 241–247, 2013
DOI: 10.2478/forma-2013-0026

DE GRUYTER OPEN
degruyter.com/view/j/forma

Formulation of Cell Petri Nets

Mitsuru Jitsukawa
Shinshu University
Nagano, Japan

Pauline N. Kawamoto
Shinshu University
Nagano, Japan

Yasunari Shidama
Shinshu University
Nagano, Japan

Summary. Based on the Petri net definitions and theorems already formalized in the Mizar article [13], in this article we were able to formalize the definition of cell Petri nets. It is based on [12]. Colored Petri net has already been defined in [11]. In addition, the conditions of the firing rule and the colored set to this definition, that defines the cell Petri nets are further extended to CPNT.i further. The synthesis of two Petri nets was introduced in [11] and in this work the definition is extended to produce the synthesis of a family of colored Petri nets. Specifically, the extension to a CPNT family is performed by specifying how to link the outbound transitions of each colored Petri net to the place elements of other nets to form a neighborhood relationship. Finally, the activation of colored Petri nets was formalized.

MSC: 68-04 03B35

Keywords: Petri net; system modelling

MML identifier: PETRI_3, version: 8.1.02 5.22.1191

The notation and terminology used in this paper have been introduced in the following articles: [1], [15], [10], [5], [6], [7], [17], [2], [3], [4], [8], [16], [13], [11], [19], [14], [18], and [9].

1. Preliminaries

Let I be a non empty set and C_1 be a many sorted set indexed by I. We say that C_1 is colored Petri net family-like if and only if

(Def. 1) Let us consider an element i of I. Then $C_1(i)$ is a colored place/transition net.

Note that there exists a many sorted set indexed by I which is colored Petri net family-like.

A colored Petri net family of I is a colored Petri net family-like many sorted set indexed by I. Let C_1 be a colored Petri net family of I and i be an element

ISSN 1426–2630(Print), 1898-9934(Online)

of I. One can check that the functor $C_1(i)$ yields a colored place/transition net. Let C_2 be a colored Petri net family of I. We say that C_2 is disjoint valued if and only if

(Def. 2) Let us consider elements i, j of I. Suppose $i \neq j$. Then

(i) the carrier of $C_2(i)$ misses the carrier of $C_2(j)$, and

(ii) the carrier' of $C_2(i)$ misses the carrier' of $C_2(j)$.

Now we state the propositions:

(1) Let us consider a set I and many sorted sets F, D, R indexed by I. Suppose

(i) for every element i such that $i \in I$ there exists a function f such that $f = F(i)$ and $\operatorname{dom} f = D(i)$ and $\operatorname{rng} f = R(i)$, and

(ii) for every elements i, j and for every functions f, g such that $i, j \in I$ and $i \neq j$ and $f = F(i)$ and $g = F(j)$ holds $\operatorname{dom} f$ misses $\operatorname{dom} g$.

Then there exists a function G such that

(iii) $G = \bigcup \operatorname{rng} F$, and

(iv) $\operatorname{dom} G = \bigcup \operatorname{rng} D$, and

(v) $\operatorname{rng} G = \bigcup \operatorname{rng} R$, and

(vi) for every elements i, x and for every function f such that $i \in I$ and $f = F(i)$ and $x \in \operatorname{dom} f$ holds $G(x) = f(x)$.

PROOF: For every element z such that $z \in \bigcup \operatorname{rng} F$ there exist elements x, y, i such that $z = \langle x, y \rangle$ and $z \in F(i)$ and $i \in I$. For every element z such that $z \in \bigcup \operatorname{rng} F$ there exist elements x, y such that $z = \langle x, y \rangle$. Reconsider $G = \bigcup \operatorname{rng} F$ as a binary relation. G is a function. For every element x, $x \in \operatorname{dom} G$ iff $x \in \bigcup \operatorname{rng} D$ by [5, (3)]. For every element x, $x \in \operatorname{rng} G$ iff $x \in \bigcup \operatorname{rng} R$ by [5, (3)]. For every elements i, x and for every function f such that $i \in I$ and $f = F(i)$ and $x \in \operatorname{dom} f$ holds $G(x) = f(x)$ by [5, (1), (3)]. □

(2) Let us consider a set I and many sorted sets Y, Z indexed by I. Suppose elements i, j. If $i, j \in I$ and $i \neq j$, then $Y(i) \cap Z(j) = \emptyset$. Then $\bigcup (Y \setminus Z) = \bigcup Y \setminus \bigcup Z$. PROOF: Set $X = Y \setminus Z$. For every element x, $x \in \bigcup \operatorname{rng} X$ iff $x \in \bigcup \operatorname{rng} Y \setminus \bigcup \operatorname{rng} Z$ by [5, (3)]. □

(3) Let us consider a set I and many sorted sets X, Y, Z indexed by I. Suppose

(i) $X \subseteq Y \setminus Z$, and

(ii) for every elements i, j such that $i, j \in I$ and $i \neq j$ holds $Y(i) \cap Z(j) = \emptyset$.

Then $\bigcup X \subseteq \bigcup Y \setminus \bigcup Z$. The theorem is a consequence of (2).

2. Synthesis of CPNT and I

Let I be a non trivial set. The functor XorDelta I yielding a non empty set is defined by the term

(Def. 3) $\{\langle i, j\rangle$, where i, j are elements of $I : i \neq j\}$.

Now we state the proposition:

(4) Let us consider a non trivial finite set I and a colored Petri net family C_2 of I. Then $\bigcup\{$(the carrier of $C_2(j))^{\mathrm{Outbds}(C_2(i))}$, where i, j are elements of $I : i \neq j\}$ is not empty.

Let I be a non trivial finite set and C_2 be a colored Petri net family of I. A connecting mapping of C_2 is a many sorted set indexed by XorDelta I and is defined by

(Def. 4) (i) rng $it \subseteq \bigcup\{$(the carrier of $C_2(j))^{\mathrm{Outbds}(C_2(i))}$, where i, j are elements of $I : i \neq j\}$, and

(ii) for every elements i, j of I such that $i \neq j$ holds $it(\langle i, j\rangle)$ is a function from Outbds$(C_2(i))$ into the carrier of $C_2(j)$.

Now we state the proposition:

(5) Let us consider colored place/transition nets C_4, C_5, a function O_1 from Outbds C_4 into the carrier of C_5, and a function q_1. Suppose

(i) dom $q_1 =$ Outbds C_4, and

(ii) for every transition t_1 of C_4 such that t_1 is outbound holds $q_1(t_1)$ is a function from the thin cylinders of the colored set of C_4 and ${}^*\{t_1\}$ into the thin cylinders of the colored set of C_4 and $O_1{}^\circ t_1$.

Then $q_1 \in (\bigcup\{$(the thin cylinders of the colored set of C_4 and $O_1{}^\circ t_1)^\alpha$, where t_1 is a transition of $C_4 : t_1$ is outbound$\})^{\mathrm{Outbds}\, C_4}$, where α is the thin cylinders of the colored set of C_4 and ${}^*\{t_1\}$.

Let I be a non trivial finite set, C_2 be a colored Petri net family of I, and O be a connecting mapping of C_2. A connecting firing rule of O is a many sorted set indexed by XorDelta I and is defined by

(Def. 5) Let us consider elements i, j of I. Suppose $i \neq j$. Then there exists a function O_2 from Outbds$(C_2(i))$ into the carrier of $C_2(j)$ and there exists a function q_2 such that $q_2 = it(\langle i, j\rangle)$ and $O_2 = O(\langle i, j\rangle)$ and dom $q_2 =$ Outbds$(C_2(i))$ and for every transition t_1 of $C_2(i)$ such that t_1 is outbound holds $q_2(t_1)$ is a function from the thin cylinders of the colored set of $C_2(i)$ and ${}^*\{t_1\}$ into the thin cylinders of the colored set of $C_2(i)$ and $O_2{}^\circ t_1$.

3. Extension to a Family of Colored Petri Nets

Let I be a non trivial finite set, C_2 be a colored Petri net family of I, O be a connecting mapping of C_2, and q be a connecting firing rule of O. Assume C_2 is disjoint valued and for every elements i, j_1, j_2 of I such that $i \neq j_1$ and $i \neq j_2$ and there exist elements x, y_1, y_2 such that $\langle x, y_1 \rangle \in q(\langle i, j_1 \rangle)$ and $\langle x, y_2 \rangle \in q(\langle i, j_2 \rangle)$ holds $j_1 = j_2$. The functor synthesis q yielding a strict colored place/transition net is defined by

(Def. 6) There exist many sorted sets P, T, S_1, T_1, C_3, F indexed by I and there exist functions U, U_1 such that for every element i of I, $P(i) =$ the carrier of $C_2(i)$ and $T(i) =$ the carrier' of $C_2(i)$ and $S_1(i) =$ the S-T arcs of $C_2(i)$ and $T_1(i) =$ the T-S arcs of $C_2(i)$ and $C_3(i) =$ the colored set of $C_2(i)$ and $F(i) =$ the firing rule of $C_2(i)$ and $U = \bigcup \operatorname{rng} F$ and $U_1 = \bigcup \operatorname{rng} q$ and the carrier of $it = \bigcup \operatorname{rng} P$ and the carrier' of $it = \bigcup \operatorname{rng} T$ and the S-T arcs of $it = \bigcup \operatorname{rng} S_1$ and the T-S arcs of $it = \bigcup \operatorname{rng} T_1 \cup \bigcup \operatorname{rng} O$ and the colored set of $it = \bigcup \operatorname{rng} C_3$ and the firing rule of $it = U {+\cdot} U_1$.

4. Definition of Cell Petri Nets

Let I be a non empty finite set and C_2 be a colored Petri net family of I. We say that C_2 is cell Petri nets if and only if

(Def. 7) There exists a function N from I into $2^{\operatorname{rng} C_2}$ such that for every element i of I, $N(i) = \{C_2(j)$, where j is an element of $I : j \neq i\}$.

Let N be a function from I into $2^{\operatorname{rng} C_2}$ and O be a connecting mapping of C_2. We say that (N, O) is cell Petri nets if and only if

(Def. 8) Let us consider an element i of I. Then $N(i) = \{C_2(j)$, where j is an element of $I : j \neq i$ and there exists a transition t of $C_2(i)$ and there exists an element s such that $\langle t, s \rangle \in O(\langle i, j \rangle)\}$.

Now we state the proposition:

(6) Let us consider a non trivial finite set I, a colored Petri net family C_2 of I, a function N from I into $2^{\operatorname{rng} C_2}$, and a connecting mapping O of C_2. Suppose

(i) C_2 is one-to-one, and

(ii) (N, O) is cell Petri nets.

Let us consider an element i of I. Then $C_2(i) \notin N(i)$.

5. Activation of Petri Nets

Let C_6 be a colored place/transition net structure. We say that C_6 has nontrivial colored set if and only if

(Def. 9) The colored set of C_6 is not trivial.

One can verify that there exists a strict colored-PT-net-like colored Petri net which has nontrivial colored set.

Let C_2 be a colored place/transition net with nontrivial colored set. One can verify that the colored set of C_2 is non trivial.

Let C_6 be a colored place/transition net with nontrivial colored set, S be a subset of the carrier of C_6, and D be a thin cylinder of the colored set of C_6 and S. A color threshold of D is a function from $\operatorname{loc} D$ into the colored set of C_6. Let C_6 be a colored place/transition net. A color count of C_6 is a function from the colored set of C_6 into $\mathbb{N}$. The colored states of C_6 yielding a non empty set is defined by the term

(Def. 10) the set of all e where e is a color count of C_6.

A colored state of C_6 is a function from C_6 into the colored states of C_6. From now on C_6 denotes a colored place/transition net with nontrivial colored set, m denotes a colored state of C_6, and t denotes an element of the carrier' of C_6.

Let C_6 be a colored place/transition net with nontrivial colored set, m be a colored state of C_6, and p be a place of C_6. Observe that the functor $m(p)$ yields a color count of C_6. Let m_1 be a color count of C_6 and x be an element. Let us observe that the functor $m_1(x)$ yields an element of $\mathbb{N}$. Let us consider C_6, m, and t. Let D be a thin cylinder of the colored set of C_6 and ${}^*\{t\}$ and C_7 be a color threshold of D. We say that t is firable on m and C_7 if and only if

(Def. 11) (i) (the firing rule of C_6)($\langle t, D\rangle$) $\neq \emptyset$, and

(ii) for every place p of C_6 such that $p \in \operatorname{loc} D$ holds $1 \leqslant m(p)(C_7(p))$.

The firable set on m and t yielding a set is defined by the term

(Def. 12) $\{D$, where D is a thin cylinder of the colored set of C_6 and ${}^*\{t\}$: there exists a color threshold C_7 of D such that t is firable on m and $C_7\}$.

Now we state the proposition:

(7) Let us consider a thin cylinder D of the colored set of C_6 and ${}^*\{t\}$. Then there exists a color threshold C_7 of D such that t is firable on m and C_7 if and only if $D \in$ the firable set on m and t.

Let us consider C_6, m, and t. Let D be a thin cylinder of the colored set of C_6 and ${}^*\{t\}$, C_7 be a color threshold of D, and p be an element of C_6. Assume t is firable on m and C_7. The Petri subtraction(C_7,m,p) yielding a function from the colored set of C_6 into $\mathbb{N}$ is defined by

(Def. 13) Let us consider an element x of the colored set of C_6. Then

(i) if $p \in \operatorname{loc} D$ and $x = C_7(p)$, then $it(x) = m(p)(x) - 1$, and

(ii) if it is not true that $p \in \operatorname{loc} D$ and $x = C_7(p)$, then $it(x) = m(p)(x)$.

Let D be a thin cylinder of the colored set of C_6 and $\overline{\{t\}}$. The Petri addition(C_7,m,p) yielding a function from the colored set of C_6 into $\mathbb{N}$ is defined by

(Def. 14) Let us consider an element x of the colored set of C_6. Then

(i) if $p \in \operatorname{loc} D$ and $x = C_7(p)$, then $it(x) = m(p)(x) + 1$, and

(ii) if it is not true that $p \in \operatorname{loc} D$ and $x = C_7(p)$, then $it(x) = m(p)(x)$.

Let D be a thin cylinder of the colored set of C_6 and $^*\{t\}$ and E be a thin cylinder of the colored set of C_6 and $\overline{\{t\}}$. Let C_{10} be a color threshold of E. The firing result(C_7,C_{10},m,p) yielding a function from the colored set of C_6 into $\mathbb{N}$ is defined by the term

(Def. 15) $$\begin{cases} \text{the Petri subtraction}(C_7,m,p), & \\ \qquad \textbf{if } t \text{ is firable on } m \text{ and } C_7, \text{ and } p \in \operatorname{loc} D \setminus \operatorname{loc} E, \\ \text{the Petri addition}(C_{10},m,p), & \\ \qquad \textbf{if } t \text{ is firable on } m \text{ and } C_7, \text{ and } p \in \operatorname{loc} E \setminus \operatorname{loc} D, \\ m(p), \qquad \textbf{otherwise}. \end{cases}$$

Let us consider a thin cylinder D_1 of the colored set of C_6 and $^*\{t\}$, a thin cylinder D_2 of the colored set of C_6 and $\overline{\{t\}}$, a color threshold C_8 of D_1, a color threshold C_9 of D_2, an element x of the colored set of C_6, and an element p of C_6. Now we state the propositions:

(8) $m(p)(x) - 1 \leqslant$ (the firing result(C_8,C_9,m,p))(x) $\leqslant m(p)(x) + 1$.

(9) If t is outbound, then $m(p)(x) - 1 \leqslant$ (the firing result(C_8,C_9,m,p))(x) $\leqslant m(p)(x)$.

Acknowledgement: We are thankful to Dr. Yatsuka Nakamura. He is the former professor of the Shinshu University. The completion of this article would not have be possible without the deep insight into the automatic proof verification system of Dr. Nakamura. Thank you.

References

[1] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.

[2] Grzegorz Bancerek. Free term algebras. *Formalized Mathematics*, 20(**3**):239–256, 2012. doi:10.2478/v10037-012-0029-6.

[3] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[4] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**): 55–65, 1990.

[6] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[7] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.

[8] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[9] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[10] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[11] Mitsuru Jitsukawa, Pauline N. Kawamoto, Yasunari Shidama, and Yatsuka Nakamura. Cell Petri net concepts. *Formalized Mathematics*, 17(**1**):37–42, 2009. doi:10.2478/v10037-009-0004-z.
[12] Pauline N. Kawamoto and Yatsuka Nakamura. *On Cell Petri Nets*. Journal of Applied Functional Analysis, 1996.
[13] Pauline N. Kawamoto, Yasushi Fuwa, and Yatsuka Nakamura. Basic Petri net concepts. *Formalized Mathematics*, 3(**2**):183–187, 1992.
[14] Krzysztof Retel. Properties of first and second order cutting of binary relations. *Formalized Mathematics*, 13(**3**):361–365, 2005.
[15] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**): 115–122, 1990.
[16] Andrzej Trybulec. Many sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.
[17] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[18] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[19] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1 (**1**):73–83, 1990.

Received December 8, 2013