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Triangular Fuzzy Set Composed of Two Intersecting Affine Maps[1]

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Summary. This article, following on from the previous article [11], formulates various properties regarding triangular membership functions. The main theorem is relationship between a triangular membership function composed of two straight lines and a MAX function and a triangular membership function defined from the horizontal axis coordinates of the vertices of the triangle. Moreover, defuzzified value [15] of triangular membership function [13], [12], [17], [7], [18], [6], construction of trapezoidal membership function using triangular membership function [9] and integration of two connected functions [14] are formalised [8], [4].

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1. Triangular Fuzzy Set

Let us consider real numbers p, q, s, t. Now we state the propositions:

(1) If $p > 0$ and $p \cdot s < 0$, then $t > q$ iff $\frac{t-q}{p-s} > 0$.

(2) If $p > 0$ and $p \cdot s < 0$, then $t < q$ iff $\frac{t-q}{p-s} < 0$. The theorem is a consequence of (1).

(3) If $p > 0$ and $p \cdot s < 0$, then $\frac{p \cdot t - q \cdot s}{p-s} < 0$ iff $-\frac{q}{p} > -\frac{t}{s}$.

(4) If $p > 0$ and $p \cdot s < 0$, then $\frac{p \cdot t - q \cdot s}{p-s} > 0$ iff $-\frac{q}{p} < -\frac{t}{s}$.

Now we state the propositions:

[1] Fuzzy Arithmetic ???

(5) Let us consider real numbers a, b. Then $a, b \in \mathbb{R} \setminus]a, b[$.

(6) Let us consider real numbers a, b, c, d. Suppose $a < b < c < d$. Then $\mathbb{R} \setminus]a, d[\cap [b, c] = \emptyset$.

Let us consider real numbers p, q, s, t. Now we state the propositions:

(7) Suppose $s \neq p$. Then

(i) $(\mathrm{AffineMap}(p, q))(\frac{t-q}{p-s}) = (\mathrm{AffineMap}(s, t))(\frac{t-q}{p-s})$, and

(ii) $(\mathrm{AffineMap}(s, t))(\frac{t-q}{p-s}) = \frac{p \cdot t - q \cdot s}{p-s}$.

(8) Suppose $p \cdot s < 0$ and $p > 0$ and $\frac{p \cdot t - q \cdot s}{p-s} > 0$. Then

(i) $-\frac{q}{p} < \frac{t-q}{p-s} < -\frac{t}{s}$, and

(ii) $-\frac{q}{p} < -\frac{t}{s}$.

Now we state the propositions:

(9) Let us consider a real number r, subsets C, U of $\mathbb{R}$, and functions f, g from $\mathbb{R}$ into $\mathbb{R}$. Then $r \cdot (f{\restriction}C {+\cdot} g{\restriction}U) = r \cdot (f{\restriction}C) {+\cdot} r \cdot (g{\restriction}U)$.
PROOF: Set $f_1 = r \cdot (f{\restriction}C {+\cdot} g{\restriction}U)$. Set $f_2 = r \cdot (f{\restriction}C) {+\cdot} r \cdot (g{\restriction}U)$. For every object x such that $x \in \mathrm{dom}\, f_1$ holds $f_1(x) = f_2(x)$ by [3, (13), (11)]. □

(10) Let us consider functions f, g, h from $\mathbb{R}$ into $\mathbb{R}$, and real numbers a, b, c. Suppose $a \leqslant b \leqslant c$ and f is continuous and g is continuous and $h{\restriction}[a, c] = f{\restriction}[a, b] {+\cdot} g{\restriction}[b, c]$ and $\int\limits_{[a,b]} f(x)dx \neq 0$ and $\int\limits_{[b,c]} g(x)dx \neq 0$ and $f(b) = g(b)$. Then $\mathrm{centroid}(h, [a, c]) = \frac{1}{\int\limits_{[a,c]} h(x)dx} \cdot (\int\limits_{[a,b]} (\mathrm{id}_{\mathbb{R}} \cdot f)(x)dx + \int\limits_{[b,c]} (\mathrm{id}_{\mathbb{R}} \cdot g)(x)dx)$.

(11) Let us consider real numbers a, b, c. Suppose $a < b < c$. Then $\mathrm{TriangularFS}(a, b, c){\restriction}[$
$(\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a})){\restriction}[a, b] {+\cdot} (\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b})){\restriction}[b, c]$.
PROOF: Set $f_5 = (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a})){\restriction}[a, b] {+\cdot} (\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b})){\restriction}[b, c]$. For every object x such that $x \in \mathrm{dom}(\mathrm{TriangularFS}(a, b, c){\restriction}[a, c])$ holds $(\mathrm{TriangularFS}(a, b, c){\restriction}[a, c])(x) = f_5(x)$ by [2, (49)], [3, (14), (13)]. □

Let us consider real numbers a, b, c, r and a function f from $\mathbb{R}$ into $\mathbb{R}$. Now we state the propositions:

(12) Suppose $a < b < c$. Then $(r \cdot \mathrm{TriangularFS}(a, b, c)){\restriction}[a, c] = (r \cdot (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}$
$(\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b}))){\restriction}[b, c]$. The theorem is a consequence of (11) and (9).

(13) Suppose $a < b < c$ and for every real number x, $f(x) = (r \cdot \mathrm{TriangularFS}(a, b, c))(x)$. Then $f{\restriction}[a, c] = (r \cdot (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a})))){\restriction}[a, b] {+\cdot} (r \cdot (\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b})))){\restriction}[b, c]$

PROOF: Set $f_5 = (r\cdot(\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a})))\restriction[a, b]+\cdot(r\cdot(\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b})))\restriction[b,$
For every object x such that $x \in \mathrm{dom}(f\restriction[a, c])$ holds $(f\restriction[a, c])(x) = f_5(x)$ by [2, (49)], (11), (9), [16, (41)]. □

Now we state the propositions:

(14) Let us consider real numbers p, q, s, t, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $p\cdot s < 0$ and $p > 0$ and $\frac{p\cdot t-q\cdot s}{p-s} > 0$. Then $(\mathrm{AffineMap}(p, q))\restriction[-\frac{q}{p}, \frac{t-q}{p-s}]+\cdot(\mathrm{Affin}$ $\frac{p\cdot t-q\cdot s}{p-s} \cdot (\mathrm{TriangularFS}((-\frac{q}{p}), \frac{t-q}{p-s}, (-\frac{t}{s}))\restriction[-\frac{q}{p}, -\frac{t}{s}])$. The theorem is a consequence of (8) and (13).

(15) Let us consider real numbers p, q, s, t, x. Suppose $p\cdot s < 0$ and $p > 0$ and $\frac{p\cdot t-q\cdot s}{p-s} > 0$ and $x \notin]-\frac{q}{p}, -\frac{t}{s}[$. Then $((\mathrm{AffineMap}(p, q))\restriction]-\infty, \frac{t-q}{p-s}]+\cdot(\mathrm{AffineMap}(s, t))$ 0. The theorem is a consequence of (8).

Let us consider real numbers a, b, c, x. Now we state the propositions:

(16) Suppose $a < b < c$ and $x \notin]a, c[$. Then $((\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}))\restriction]-\infty, b]+\cdot(\mathrm{Affine}$ 0. The theorem is a consequence of (15).

(17) If $a < b < c$ and $x \in [a, c]$, then $(\mathrm{TriangularFS}(a, b, c))(x) \geqslant 0$.

Now we state the propositions:

(18) Let us consider real numbers a, b, c, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $a < b < c$ and $f = (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}))\restriction]-\infty, b]+\cdot(\mathrm{AffineMap}(-\frac{1}{c-b}, \frac{c}{c-b}))\restriction[$ Then $\max_+(f) = \mathrm{TriangularFS}(a, b, c)$.

PROOF: For every object x such that $x \in \mathrm{dom}(\mathrm{TriangularFS}(a, b, c))$ holds $(\mathrm{TriangularFS}(a, b, c))(x) = (\max_+(f))(x)$ by [2, (49)], [10, (20)], (11), (17). □

(19) Let us consider real numbers a, b, c, r, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $a < b < c$ and $r > 0$ and $f = r\cdot((\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}))\restriction]-\infty, b]+\cdot(\mathrm{AffineM}$ Then $\max_+(f) = r \cdot \mathrm{TriangularFS}(a, b, c)$.

PROOF: For every object x such that $x \in \mathrm{dom}(r \cdot \mathrm{TriangularFS}(a, b, c))$ holds $(r \cdot \mathrm{TriangularFS}(a, b, c))(x) = (\max_+(f))(x)$ by [1, (74), (65), (43)], (18). □

(20) Let us consider real numbers p, q, s, t, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $p \cdot s < 0$ and $p > 0$ and $\frac{p\cdot t-q\cdot s}{p-s} > 0$ and for every real number x, $f(x) = ((\mathrm{AffineMap}(p, q))\restriction]-\infty, \frac{t-q}{p-s}]+\cdot(\mathrm{AffineMap}(s, t))\restriction[\frac{t-q}{p-s}, +\infty[)(x)$. Then $\max_+(f) = \frac{p\cdot t-q\cdot s}{p-s} \cdot \mathrm{TriangularFS}((-\frac{q}{p}), \frac{t-q}{p-s}, (-\frac{t}{s}))$.

PROOF: Set $r = \frac{p\cdot t-q\cdot s}{p-s}$. For every object x such that $x \in \mathrm{dom}(\max_+(f))$ holds $(\max_+(f))(x) = (r \cdot \mathrm{TriangularFS}((-\frac{q}{p}), \frac{t-q}{p-s}, (-\frac{t}{s})))(x)$ by (8), [3, (11)], [2, (49)], [3, (13)]. □

(21) Let us consider real numbers a, b, c, r, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $a < b < c$ and for every real number x, $f(x) = (r \cdot$

TriangularFS$(a,b,c))(x)$. Then $\int\limits_{[a,c]} f(x)dx = \frac{r\cdot(c-a)}{2}$. The theorem is a consequence of (13).

(22) Let us consider real numbers a, b, c. Suppose $a<b<c$. Then integral TriangularFS($\frac{c-a}{2}$. The theorem is a consequence of (21).

(23) Let us consider real numbers a, b, c, r, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Suppose $a<b<c$ and $r\neq 0$ and for every real number x, $f(x) = (r\cdot\text{TriangularFS}(a,b,c))(x)$. Then centroid$(f,[a,c]) = \frac{a+b+c}{3}$. The theorem is a consequence of (13), (10), and (21).

(24) Let us consider real numbers p, q, s, t, and functions f, F from $\mathbb{R}$ into $\mathbb{R}$. Suppose $p\cdot s<0$ and $p>0$ and $\frac{p\cdot t-q\cdot s}{p-s}>0$ and $f = (\text{AffineMap}(p,q))\restriction]-\infty,\frac{t-q}{p-s}]+\cdot$(A and $F=\max_{+}(f)$. Then centroid$(F,[-\frac{q}{p},-\frac{t}{s}]) = \frac{-\frac{q}{p}+\frac{t-q}{p-s}+-\frac{t}{s}}{3}$.
PROOF: $-\frac{q}{p} < \frac{t-q}{p-s} < -\frac{t}{s}$. For every real number x, $F(x) = (\frac{p\cdot t-q\cdot s}{p-s}\cdot$ TriangularFS$((-\frac{q}{p}),\frac{t-q}{p-s},(-\frac{t}{s})))(x)$ by [10, (12)], (20). □

Let us consider a non empty, closed interval subset A of $\mathbb{R}$, real numbers p, q, r, s, t, u, and a function f from $\mathbb{R}$ into $\mathbb{R}$. Now we state the propositions:

(25) Suppose $p\cdot s<0$ and $p>0$ and for every real number x, $f(x) = \max(r,\min(u,((\text{AffineMap}(p,q))\restriction]-\infty,\frac{t-q}{p-s}]+\cdot(\text{AffineMap}(s,t))\restriction[\frac{t-q}{p-s},+\infty[)(x)))$. Then f is Lipschitzian.
PROOF: Set $F = (\text{AffineMap}(p,q))\restriction]-\infty,\frac{t-q}{p-s}]+\cdot(\text{AffineMap}(s,t))\restriction[\frac{t-q}{p-s},+\infty[$. Consider r_2 being a real number such that $0<r_2$ and for every real numbers x_1, x_2 such that $x_1, x_2\in\operatorname{dom}F$ holds $|F(x_1)-F(x_2)|\leqslant r_2\cdot|x_1-x_2|$. There exists a real number r_1 such that $0<r_1$ and for every real numbers x_1, x_2 such that $x_1, x_2\in\operatorname{dom}f$ holds $|f(x_1)-f(x_2)|\leqslant r_1\cdot|x_1-x_2|$ by [1, (74), (65), (43)]. □

(26) Suppose $p\cdot s<0$ and $p>0$ and for every real number x, $f(x) = \max(r,\min(u,((\text{AffineMap}(p,q))\restriction]-\infty,\frac{t-q}{p-s}]+\cdot(\text{AffineMap}(s,t))\restriction[\frac{t-q}{p-s},+\infty[)(x)))$. Then

(i) f is integrable on A, and

(ii) $f\restriction A$ is bounded.

The theorem is a consequence of (25).

2. Trapezoidal Fuzzy Set

Let us consider real numbers a, b, c, d and an object x. Now we state the propositions:

(27) If $a < b < c < d$, then if $x \in [a, b]$, then $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}))(x)$.
PROOF: For every object x such that $x \in [a, b]$ holds $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(\frac{1}{b-a}, -\frac{a}{b-a}))(x)$ by (6), [3, (13)], [2, (49)], [3, (11), (15)]. □

(28) If $a < b < c < d$, then if $x \in [b, c]$, then $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(0, 1))(x)$.
PROOF: For every object x such that $x \in [b, c]$ holds $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(0, 1))(x)$ by [3, (11)], [2, (49)], (5), [3, (13), (15)]. □

(29) If $a < b < c < d$, then if $x \in [c, d]$, then $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(-\frac{1}{d-c}, \frac{d}{d-c}))(x)$.
PROOF: For every object x such that $x \in [c, d]$ holds $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = (\mathrm{AffineMap}(-\frac{1}{d-c}, \frac{d}{d-c}))(x)$ by [3, (13)], [2, (49)]. □

Now we state the propositions:

(30) Let us consider real numbers a, b, c, d. Suppose $a < b < c < d$. Let us consider a real number x. If $x \notin [a, d]$, then $(\mathrm{TrapezoidalFS}(a, b, c, d))(x) = 0$.
PROOF: For every real number x such that $x \notin [a, d]$ holds $(\mathrm{TrapezoidalFS}(a, b, c, d))($
0 by [3, (11)], [2, (49)]. □

(31) Let us consider real numbers a, b, c, d. Suppose $a < b < c < d$. Then $\mathrm{TriangularFS}(a, b, c) + \mathrm{TriangularFS}(b, c, d) = \mathrm{TrapezoidalFS}(a, b, c, d)$.
PROOF: For every object x such that $x \in \mathrm{dom}(\mathrm{TrapezoidalFS}(a, b, c, d))$ holds $(\mathrm{TriangularFS}(a, b, c) + \mathrm{TriangularFS}(b, c, d))(x) = (\mathrm{TrapezoidalFS}(a, b, c, d))(x$
by [11, (48), (47)], (29), [11, (46)]. □

3. Related Properties about the Integral of Two Connected Functions

From now on A denotes a non empty, closed interval subset of $\mathbb{R}$.

Let us consider real numbers a, b, c and functions f, g from $[a, c]$ into $\mathbb{R}$. Now we state the propositions:

(32) If $a \leqslant b \leqslant c$, then $f \restriction [a, b] {+\cdot} g \restriction [b, c]$ is a function from $[a, c]$ into $\mathbb{R}$.

(33) Suppose $a \leqslant b \leqslant c$ and $f \restriction [a, c]$ is bounded and $g \restriction [a, c]$ is bounded. Then $(f \restriction [a, b] {+\cdot} g \restriction [b, c]) \restriction [a, c]$ is bounded.
PROOF: Set $h = f \restriction [a, b] {+\cdot} g \restriction [b, c]$. There exists a real number r such that for every set y such that $y \in \mathrm{dom}(h \restriction [a, c])$ holds $|(h \restriction [a, c])(y)| < r$ by [3, (11)], [2, (49)], [19, (62)], [3, (13)]. □

Now we state the proposition:

(34) Let us consider a real number a, and functions f, g from $\mathbb{R}$ into $\mathbb{R}$. Suppose $f \restriction A$ is bounded and $g \restriction A$ is bounded and $a \in A$. Then

(i) $(f{\restriction}]-\infty, a[+\cdot g{\restriction}[a, +\infty[){\restriction}[\inf A, a]$ is bounded, and

(ii) $(f{\restriction}]-\infty, a[+\cdot g{\restriction}[a, +\infty[){\restriction}[a, \sup A]$ is bounded.

PROOF: Set $F = f{\restriction}]-\infty, a[+\cdot g{\restriction}[a, +\infty[$. Set $L_9 = [\inf A, a]$. Set $a_{-13} = [a, \sup A]$. There exists a real number r such that for every set y such that $y \in \mathrm{dom}(F{\restriction}L_9)$ holds $|(F{\restriction}L_9)(y)| < r$ by [19, (57)], [5, (4)], [2, (49)], [3, (11), (13)]. There exists a real number r such that for every set y such that $y \in \mathrm{dom}(F{\restriction}a_{-13})$ holds $|(F{\restriction}a_{-13})(y)| < r$ by [19, (57)], [5, (4)], [2, (49)], [3, (11), (13)]. □

Let us consider a real number a and functions f, g, h from $\mathbb{R}$ into $\mathbb{R}$. Now we state the propositions:

(35) Suppose $f{\restriction}A$ is bounded and f is integrable on A and $g{\restriction}A$ is bounded and g is integrable on A and $a \in A$ and $h = f{\restriction}]-\infty, a]+\cdot g{\restriction}[a, +\infty[$ and $f(a) = g(a)$. Then

(i) h is integrable on $[\inf A, a]$, and

(ii) h is integrable on $[a, \sup A]$.

PROOF: For every object x such that $x \in \mathrm{dom}(f{\restriction}[\inf A, a])$ holds $(f{\restriction}[\inf A, a])(x) = (h{\restriction}[\inf A, a])(x)$ by [2, (49)], [3, (15)]. For every object x such that $x \in \mathrm{dom}(g{\restriction}[a, \sup A])$ holds $(g{\restriction}[a, \sup A])(x) = (h{\restriction}[a, \sup A])(x)$ by [2, (49)], [3, (13)]. □

(36) Suppose $f{\restriction}A$ is bounded and f is integrable on A and $g{\restriction}A$ is bounded and g is integrable on A and $a \in A$ and $h = f{\restriction}]-\infty, a]+\cdot g{\restriction}[a, +\infty[$ and $f(a) = g(a)$. Then $\int_A h(x)dx = \int_{[\inf A, a]} f(x)dx + \int_{[a, \sup A]} g(x)dx$.

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